the picture that the propagation of EM waves become considerably more complex and interesting.

- Whereas pond ripples are confined to the water surface and therefore propagate in only two dimensions, EM waves propagate in three dimensional space, like sound waves (imagine if you could only hear your stereo when you put your ear on the floor!).

- Whereas the transverse oscillations of water particles in pond ripples are constrained to be vertical, there is no similar constraint on the electric field oscillations in EM waves. The orientation of the electric field vector may lie in any direction, as long as it is in the plane normal to the direction of propagation. Occasionally it is necessary to pay attention to this orientation, in which case we refer to the *polarization* of the wave as vertical, horizontal, or some other direction. A more detailed discussion of polarization will be taken up in Section 2.3

2.2 Frequency

Up until now, we have imagined an arbitrary EM disturbance and given no thought to its detailed dependence in time. In principle, we could assume any kind of EM disturbance we like — a lightning discharge, a refrigerator magnet dropping to the kitchen floor, the radiation emitted by a radio tower, or a supernova explosion in deep space. In each case, Maxwell's equations would describe the propagation of the resulting EM disturbance equally well.

Let's consider a special case, however. Imagine that we take our magnet and place it on a steadily rotating turntable. The fluctuations in the magnetic field (and in the associated electric field) are now periodic. The frequency of the fluctuations measured at a distance by a stationary detector will be the same as the frequency of rotation ν of the turntable. But recall that the periodic disturbance propagates outward not instantaneously but at the fixed speed of light c. The distance λ that the fluctuation propagates during one cycle of the turntable is called the *wavelength* and is given by

$$\boxed{\lambda = \frac{c}{\nu}\,.} \tag{2.1}$$

In the above thought experiment, ν is extremely low (order 1 sec^{-1}) and the wavelength is therefore extremely large (order 10^5 km).

In nature, electromagnetic waves can exist with an enormous range of frequencies, from a few cycles per second or even far less to more than 10^{26} cyles per second in the case of extremely energetic gamma waves produced by nuclear reactions. According to (2.1), EM wavelengths can thus range from hundreds of thousands of kilometers or more to less than the diameter of an atomic nucleus.

Because it is a common point of confusion, it bears emphasizing that the wavelength is *not* a measure of how *far* an EM wave can propagate. In a vacuum, that distance is always infinite, regardless of wavelength. In a medium such as water or air, wavelength *does* matter, but in a rather indirect and highly complex way.

Problem 2.1:

(a) Visible light has a wavelength of approximately 0.5 μm. What is its frequency in Hz?

(b) Weather radars typically transmit EM radiation with a frequency of approximately 3 GHz (GHz = "Gigahertz" = 10^9 Hz). What is its wavelength in centimeters?

(c) In the U.S., standard AC electrical current has a frequency of 60 Hz. Most machinery and appliances that use this current, as well as the power lines that transport the electric power, emit radiation with this frequency. What is its wavelength in km?

Problem 2.2:

Whenever we talk about a single frequency ν characterizing an electromagnetic wave, we are tacitly assuming that the source and the detector are stationary relative to one another, in which case ν is indeed the same for both. However, if the distance between the two is changing with velocity v (positive v implying increasing separation), then the frequency of radiation ν_1 emitted by the source will be different than the frequency ν_2 observed by the detector. In particular, the frequency shift $\Delta\nu = \nu_1 - \nu_2$ is approximately proportional to v, a phenomenon known as *Doppler shift*.

(a) Derive the precise relationship between $\Delta\nu$ and v, by considering the time Δt elapsed between two successive wave crests reaching the detector with speed c.

(b) For the case that $v \ll c$, show that your solution to (a) simplifies to a proportionality between $\Delta\nu$ and v.

2.2.1 Frequency Decomposition

The above discussion of periodic waves is interesting, but what does it have to do with real EM radiation? After all, the EM disturbance that arises from a lightning discharge, or from dropping a magnet on the floor, is clearly not a steadily oscillating signal but more likely a short, chaotic pulse! What sense does it make to speak of a specific frequency or wavelength in these cases?

The answer is that any arbitrary EM fluctuation, short or long, can be thought of as a *composite* of a number (potentially infinite) of different "pure" periodic fluctuations. Specifically, any continuous function of time $f(t)$ can be expressed as a sum of pure sine functions as follows:

$$f(t) = \int_0^\infty \alpha(\omega)\sin[\omega t + \phi(\omega)]\, d\omega \tag{2.2}$$

where $\alpha(\omega)$ is the amplitude of the sine function contribution for each specific value of the angular frequency ω and $\phi(\omega)$ gives the corresponding phase. If $f(t)$ itself is already a pure sine function $\sin(\omega_0 t + \phi_0)$, then of course $\alpha = 0$ for all values of ω except ω_0. For more general functions $f(t)$, $\alpha(\omega)$ and $\phi(\omega)$ may be quite complicated. It is beyond the scope of this book to explain *how* we find $\alpha(\omega)$ etc. for any given $f(t)$; it is only important to recognize that it can, in principle, always be done.[1]

[1]This so-called *Fourier decomposition* is extremely useful throughout the physical and engineering sciences, including other areas of atmospheric dynamics and climatology, so if you haven't seen anything like this before, it is certainly worth taking the time to read up on it.

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