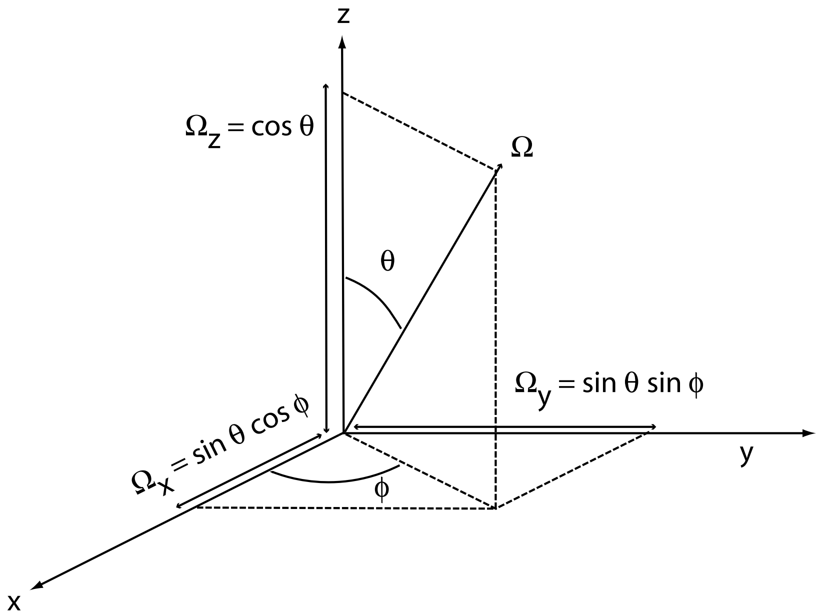

Fig. 2.3: The relationship between Cartesian and spherical coordinates.

a reference point on the horizon, so that $0 < \phi < 2\pi$. It is not usually terribly important which point of the compass is used as the reference in any given application, and it is often chosen to be whatever is most natural for the problem at hand, such as the direction of the sun.

Any possible direction above or below the horizon may thus be described via the two angles θ and ϕ. Sometimes directions may be expressed abstractly in terms of a unit vector $\hat{\mathbf{\Omega}}$, in which case no particular coordinate system is implied. Thus the same direction $\hat{\mathbf{\Omega}}$ could be represented by $[\sqrt{2}/2, 0, \sqrt{2}/2]$ in (x, y, z)-coordinates, by $[\pi/4, 0]$ in $(\theta, \phi))$-coordinates when $\theta = 0$ defines the vertical direction, or even by $[0, 0]$ if $\theta = 0$ should for some reason be chosen to coincide with the direction of the sun at a time when the latter is 45° above the horizon.

Return to: A First Course in Atmospheric Radiation

Solid Angle

Another essential concept is that of *solid angle*. Surprisingly many new students of atmospheric radiation find this concept confusing, presumably because they haven't had occasion to consciously use it before, unlike the angles we have been measuring with protractors since third grade. But it's really very simple: *solid angle is to "regular" angle as area is to length.* You can think of solid angle as something you might measure in "square radians" or "square degrees", except the actual unit used is called the *steradian*. We will give a precise definition of this unit later.

In absolute terms, the sun has a certain diameter in kilometers and a certain cross-sectional area in km^2. But absolute dimensions are often of secondary importance in radiative transfer, compared with *angular* dimensions, which describe how big an object or source of radiation *looks* from a particular vantage point. Thus, what matters for solar radiation reaching the earth is that the sun's disk has a particular angular diameter in units of degrees or radians, and it also subtends (or presents) a certain solid angle in units of steradians. Solid angle is thus a measure of how much of your visual field of view is occupied by an object. For example, the sun subtends a much larger solid angle as viewed from the planet Mercury than it does from Earth. Also, from our perspective here on Earth, the full moon subtends nearly the same solid angle as the Sun, even though the latter body is much larger in absolute terms. A half moon, of course, subtends half the solid angle of the full moon.

Definition of Steradian

Now that you understand what solid angle *is*, you can appreciate a simple definition of the unit *steradian* (abbreviation sr). Imagine you are at the center of a sphere of unit radius — it doesn't matter whether the unit is a kilometer, a mile, a furlong or what have you. The total surface area of the sphere is 4π square units. *Likewise, the combined solid angle represented by every direction you can possibly look is 4π steradians.* The surface area of just one half of the sphere is 2π units squared. *Likewise, the entire sky above the horizon* (or "celestial dome") *subtends a solid angle of 2π steradians, as does (separately) the lower hemisphere of your field of vision, representing everything below the*

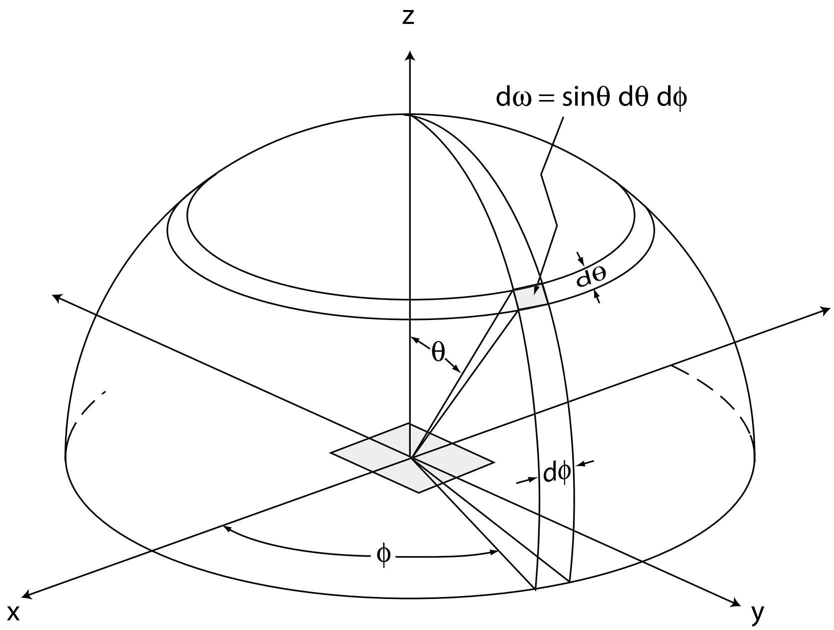

Fig. 2.4: The relationship between solid angle and polar coordinates.

horizon.

Conceptually, you can determine the solid angle subtended by any object by tracing its outline on your unit sphere and then measuring the actual surface area of the tracing. (Note the analogy to radians as a measure of arc length on the unit circle.) This is generally not a practical approach, however, so we instead invoke our polar coordinate system so as to be able to define an infinitesimal increment of solid angle as follows:

$$\boxed{d\omega = \sin\theta \; d\theta \; d\phi \; .} \tag{2.45}$$

This relationship is depicted schematically in Fig. 2.4.

In other words, if you paint an infinitesimal rectangle on the surface of your unit sphere, and it has angular dimensions $d\theta$ (zenith angle) and $d\phi$ (azimuth angle) and is positioned at θ, then the above expression gives you the increment of solid angle subtended. Why does $\sin\theta$ appear in there? Simple: for the same reason that a 1°