



Fig. 2.5: Geometric framework for computing the solid angle subtended by a sphere of radius  $R$  whose center is a distance  $D$  from the observer.

more brightly than it does an animal lurking in those bushes at the edge of the campsite. Likewise, the earth is much more brightly illuminated by the sun than is Pluto.

You might hastily infer from these observations that the *intensity* of radiation associated with a given point at the source is also a function of the distance of the observer. If you were talking about the incident *flux*, you would be right. This is not the case for intensity however. On the contrary, *in a vacuum or other transparent medium without reflecting surfaces, radiant intensity is conserved along*

any optical path.

To verify this principle in a simple case, tape a sheet of white paper to the wall at eye level. Look at it from a couple feet away; then back up and look at it from across the room. Although its apparent size (solid angle) changes with distance, its apparent brightness does not (assuming you didn't adjust the room lights!).

**Problem 2.14:** Verify the above principle for the Sun's disk by deriving a formula for its average intensity as seen from a distance  $D$  from the Sun's center, given that its total radiant power output is  $P$  and its radius is  $R_s$ . For simplicity, assume that the Sun's intensity  $I$  is the same at all points on the visible disk (this is not strictly true). You will need to derive an exact expression for the solid angle subtended by the Sun's disk for arbitrary  $D > R_s$ . If done correctly, your final solution for  $I$  should not depend on  $D$ . (Hint: To find the solid angle subtended by the Sun, assume that it is directly overhead, so that the center coincides with the  $z$ -axis and the edge is defined by  $\theta = \theta_{\max}$  (see Fig. 2.5). Then integrate (2.45) with the appropriate limits.)

The same principle applies to optical systems such as lenses, mirrors, prisms, etc., as long as we ignore losses due to absorption and/or partial reflections (e.g., from the surface of lenses). A magnifying glass makes objects appear larger, but it has no effect on the object's brightness. The same is true of binoculars and telescopes.<sup>4</sup>

### Intensity and Polarization<sup>†</sup>

For many applications, one need only keep track of the total *scalar* intensity  $I$  of a stream of radiation as defined above. However, we

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<sup>4</sup>But wait, you say. Telescopes definitely do make stars appear brighter: many that are invisible to the naked eye become clearly visible through a telescope. How can this behavior be reconciled with the previous assertions? The explanation is that stars subtend a angle far too small for the eye to resolve. As a result, the eye responds not to the intensity of the star but rather to the total flux integrated over a finite solid angle. That solid angle is determined by the eye's resolving power. Thus, moderately near-sighted individuals will see a few bright stars but will miss many more that are easily detectable by sharp-eyed people. A telescope increases the solid angle subtended by a star, and thus the total flux from that direction, making it more easily visible to everyone.

previously alluded to the polarization properties of EM radiation, and it is sometimes necessary to keep track of these as well. One reason might be a need for greater accuracy in radiative transfer calculations, as disregarding polarization almost always entails an approximation, especially when scattering by particles or surfaces is important. Another occasion arises when you are measuring radiation of one particular polarization (e.g. linear vertical or horizontal). This is often the case for microwave remote sensing instruments.

When polarization must be considered, we require a representation of intensity that is capable of providing complete information about the state of polarization. One such representation gives the intensity as Intensity!as Stokes vector a four-element vector

$$\vec{\mathbf{I}} = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}. \quad (2.48)$$

The elements of this vector are called the *Stokes parameters*. The first element,  $I$ , is the same as the scalar intensity we have already discussed. The remaining elements  $Q$ ,  $U$ ,  $V$  contain information concerning the *degree* of polarization (recall that incoherent radiation can be polarized to any degree, whereas coherent radiation is always fully polarized), about the *preferred orientation* of the polarization, and about the *nature* of the polarization – circular, linear, or something in between. In particular, the *degree of polarization* is defined as  $\sqrt{Q^2 + U^2 + V^2}/I$ . The ratios  $\sqrt{Q^2 + U^2}/I$  and  $V/I$ , respectively, are *the degree of linear polarization* and *the degree of circular polarization*.

Thus, for completely unpolarized radiation,  $Q = U = V = 0$ , and for fully polarized radiation

$$I^2 = Q^2 + U^2 + V^2. \quad (2.49)$$

It is beyond the scope of this introductory text to give a detailed electromagnetic definition of each of the Stokes parameters.<sup>5</sup> However, some illustrative examples are given in Table 2.1.

<sup>5</sup>A good overview is given in Section 2.3 of S94.