contribution arriving from a very small element of solid angle $d\omega$ centered on a given direction of propagation $\hat{\mathbf{\Omega}}$. It follows that the flux incident on, passing through, or emerging from an arbitrary surface is given by an integral over the relevant range of solid angle of the intensity.

Let us start by considering the flux emerging *upward* from a horizontal surface: it must be an integral of the intensity $I(\hat{\mathbf{\Omega}})$ over all possible directions $\hat{\mathbf{\Omega}}$ directed skyward; i.e., into the 2π steradians of solid angle corresponding to the upper hemisphere. There is one minor complication, however. Recall that intensity is defined in terms of flux per unit solid angle *normal to the beam*. For our horizontal surface, however, only one direction is normal; radiation from all other directions passes through the surface at an oblique angle (Fig. 2.6). Thus, we must weight the contributions to the flux by the cosine of the incidence angle relative to the normal vector $\hat{\mathbf{n}}$. For the upward-directed flux $F^{\uparrow}$, we therefore have the following relationship:

$$F^{\uparrow} = \int_{2\pi} I^{\uparrow}(\hat{\mathbf{\Omega}})\hat{\mathbf{n}} \cdot \hat{\mathbf{\Omega}}\, d\omega. \tag{2.53}$$

The above expression is generic: it doesn't depend on one's choice of coordinate system. In practice, it is convenient to again introduce spherical polar coordinates, with the z-axis normal to the surface:

$$\boxed{F^{\uparrow} = \int_0^{2\pi}\int_0^{\pi/2} I^{\uparrow}(\theta,\phi)\cos\theta\sin\theta\; d\theta d\phi\;,} \tag{2.54}$$

where we have used (2.45) to express $d\omega$ in terms of θ and ϕ.

For the downward flux, we integrate over the lower hemisphere, so we have

$$\boxed{F^{\downarrow} = -\int_0^{2\pi}\int_{\pi/2}^{\pi} I^{\downarrow}(\theta,\phi)\cos\theta\sin\theta\; d\theta d\phi\;.} \tag{2.55}$$

Since I is always positive, the above definitions always yield positive values for $F^{\uparrow}$ and $F^{\downarrow}$.

Key fact: For the special case that the intensity is *isotropic* — that is, I is a constant for all directions in the hemisphere, then the above integrals can be evaluated to yield

$$\boxed{F = \pi I\ .} \tag{2.56}$$

Key fact: The *net flux* is defined as the difference between upward- and downward-directed fluxes:

$$\boxed{F_{\text{net}} \equiv F^{\uparrow} - F^{\downarrow}\ ,} \tag{2.57}$$

which can be expanded as

$$F_{\text{net}} = \int_0^{2\pi}\int_0^{\pi} I(\theta,\phi)\cos\theta\ d\theta d\phi = \int_{4\pi} I(\hat{\mathbf{\Omega}})\hat{\mathbf{n}}\cdot\hat{\mathbf{\Omega}}\ d\omega. \tag{2.58}$$

Note, by the way, that the notation used throughout this subsection implies that we are relating a *broadband intensity* to a *broadband flux*. Identical relationships hold between the *monochromatic intensity* I_λ and the monochromatic fluxes $F_\lambda^{\uparrow}$ and $F_\lambda^{\downarrow}$.

Problem 2.16: If the intensity of radiation incident on a surface is uniform from all directions and denoted by the constant I, verify that the total flux is πI, as stated by (2.56). Note that this approximately describes the illumination of a horizontal surface under a heavily overcast sky. It also describes the relationship between the flux and intensity of radiation *leaving* a surface, if that surface is emitting radiation of uniform intensity in all directions.

Problem 2.17: Compute the flux from an overhead spherical sun, as seen from a planet in an orbit of radius D, given that the sun has radius R_s and a uniform intensity I_s. Make no assumptions about the size of D relative to R_s. Use two different methods for your calculation:

(a) Method 1: Integrate the intensity over the solid angle subtended by the sun, with the usual cosine-weighting relative to the local vertical.

Return to: A First Course in Atmospheric Radiation

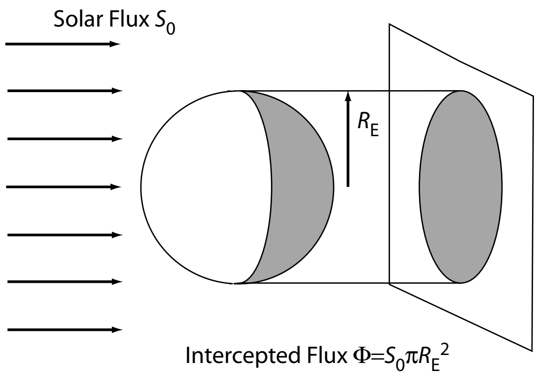

Fig. 2.7: The total flux of solar radiation intercepted by the earth is equal to the product of the incident flux density S_0 and the area of the earth's shadow.

(b) Method 2: Compute the flux density emerging from the surface of the sun, translate that into a total power emitted by the sun, and then distribute that power over the surface of a sphere of radius D.

Do your two solutions agree?

2.8 Applications to Meteorology, Climatology, and Remote Sensing

Of fundamental importance to the global climate is the input of energy from the sun and its spatial and temporal distribution. This input is a function of two variables: 1) the flux of solar radiation incident on the top of the atmosphere, and 2) the fraction of that flux that is absorbed by either the surface or the atmosphere at each point in the earth-atmosphere system. The second of these depends in a complex way on distributions of clouds and absorbing gases in the atmosphere, as well as on the absorbing properties of the surface. These are all issues that will be taken up in the remainder of this book. The first variable, however, can already be understood in terms of the material presented in this chapter.

Return to: A First Course in Atmospheric Radiation