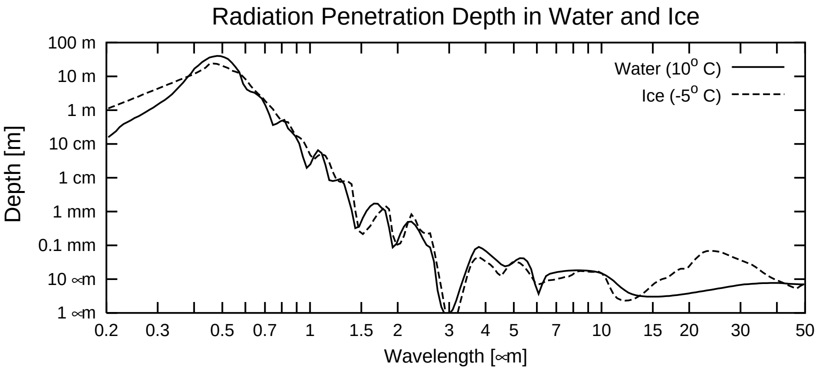

Fig. 4.2: The penetration depth of radiation in water and ice.

4.1.3 The Dielectric Constant†

In some situations, it is convenient to utilize the complex *dielectric constant* ϵ of a medium (also known as the *relative permittivity*) rather than the complex index of refraction N to describe the absorption and refraction properties of a substance at a particular wavelength. There is no need to be dismayed by the introduction of yet another parameter, as N and ϵ are very closely related.[2] In fact, for nonmagnetic materials ($\mu = \mu_0$),

$$\epsilon = \frac{\varepsilon}{\varepsilon_0} = N^2, \tag{4.6}$$

where the permittivity of the medium ε and the permittivity of free space ε_0 were previously introduced in connection with (2.36). Please keep in mind that, despite the name, the dielectric "constant" is really a *function* of frequency and, to a lesser extent, of variables like temperature and pressure.

Expanding in terms of the real and imaginary components, we find

$$\epsilon = (n_r + n_i i)^2 = n_r^2 + 2n_i n_r i - n_i^2, \tag{4.7}$$

so that the real and imaginary parts of ϵ are given by

$$\epsilon' = \mathrm{Re}\{\epsilon\} = n_r^2 - n_i^2, \qquad \epsilon'' = \mathrm{Im}\{\epsilon\} = 2n_r n_i. \tag{4.8}$$

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[2] Specifically, $\epsilon \equiv \varepsilon/\varepsilon_0$, which appears in the definition of N given by (2.36).

Though requiring somewhat more involved algebra, one may find analogous expressions for n_r and n_i in terms of ϵ' and ϵ'':

$$n_r = \sqrt{\frac{\sqrt{\epsilon'^2 + \epsilon''^2} + \epsilon'}{2}} \tag{4.9}$$

$$n_i = \sqrt{\frac{\sqrt{\epsilon'^2 + \epsilon''^2} - \epsilon'}{2}} \tag{4.10}$$

Of course, in any programming environment (e.g., Fortran) that provides for manipulation of complex numbers, there is no need to bother with the above equations; rather, one just computes $N = \sqrt{\epsilon}$ or $\epsilon = N^2$.

I introduce the dielectric constant here because, for certain types of electrodynamic computations, it is more convenient than the index of refraction. The subject of the next section is a good example.

4.1.4 Optical Properties of Heterogeneous Mixtures†

At the beginning of this chapter, we stated that we would focus on radiative interactions with media that are homogeneous *on the scales of the wavelength of the radiation*. It is possible for even a decidedly heterogeneous mixture of different substances to satisfy this criterion if the particles of each material are much smaller than the wavelength.

For example, a layer of fallen snow consists of loosely aggregated ice crystals having dimensions of approximately 1 mm. Sunlight falling on snowpack has wavelengths small enough to be affected by the small-scale structure of the snow. If we look closely, we are able to resolve individual snow crystals with our eyes. But if microwave radiation having wavelengths of 10 cm or more encounters the same snowpack, it is no more influenced by the individual particles of snow than an ocean wave crashing on the beach "cares" that the beach is not a solid surface but rather made up of grains of sand. Thus, in the microwave band, the snowpack behaves more nearly like one that is homogeneous and therefore subject to the same principles discussed in this chapter. Likewise, even complex structures like falling snowflakes are sometimes be treated as equivalent homogeneous particles for the purpose of computing radar backscatter.

Several formulas have been derived for computing the *effective index of refraction or dielectric constant* of such mixtures. The precise form of these relationships depends on what assumptions are made concerning the structure of, and relationship between, the constituents. In a two-component mixture, one component may be viewed as the *matrix* and the other as disconnected *inclusions* embedded within that matrix. For example, a cloud is best viewed as water inclusions (droplets) embedded in a matrix of air, whereas foam patches on a stormy ocean surface consist of air inclusions (bubbles) in a matrix of sea water. In other cases, such as snowpack, it may be difficult to decide whether the ice should be considered the matrix or the inclusion, because neither the air nor the ice tends to exist in isolated, disconnected pockets.

One common formula used for computing effective dielectric constants of heterogeneous mixtures is the Maxwell Garnett formula

$$\epsilon_{\mathrm{av}} = \epsilon_m \left[1 + \frac{3f\left(\frac{\epsilon-\epsilon_m}{\epsilon+2\epsilon_m}\right)}{1 - f\left(\frac{\epsilon-\epsilon_m}{\epsilon+2\epsilon_m}\right)} \right], \qquad (4.11)$$

where ϵ_m and ϵ are the dielectric constants of the matrix and the inclusions, respectively, and f is the volume fraction occupied by the inclusions.

Another widely used relationship is that of Bruggeman:

$$f_1 \frac{\epsilon_1 - \epsilon_{\mathrm{av}}}{\epsilon_1 + 2\epsilon_{\mathrm{av}}} + (1 - f_1)\frac{\epsilon_2 - \epsilon_{\mathrm{av}}}{\epsilon_2 + 2\epsilon_{\mathrm{av}}} = 0, \qquad (4.12)$$

where ϵ_1 and ϵ_2 are the dielectric constants of the two components, and f_1 is the volume fraction occupied by component 1. Note that this formula treats both components symmetrically; it does not distinguish between matrix and inclusion.

It is straightforward to extend either of the above formulas to mixtures of three or more components. This is desirable in the treatment of radar backscatter by melting snowflakes, for example, which may be regarded as mixtures of air, ice, and liquid water. In the case of the Maxwell Garnett formula, one first computes ϵ_{av} for two components, choosing one to be matrix and the other inclusion. The result may then be combined with a third component by