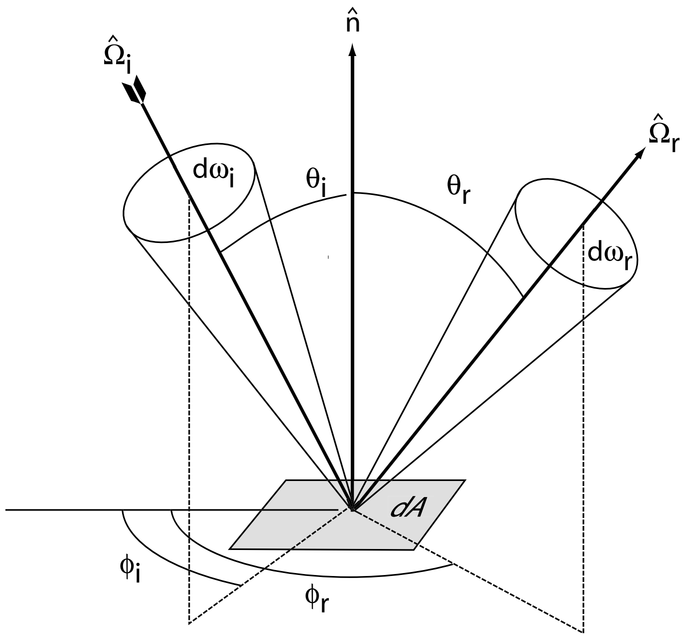

Fig. 5.4: Geometry and relevant variables for the definition of the BDRF.

where $\hat{\mathbf{n}}$ is the vertical unit vector, so that $\hat{\mathbf{n}} \cdot \hat{\mathbf{\Omega}}_i = \cos\theta_i$.

Recall that the integral as written above represents a generic integration over solid angle covering the entire upper hemisphere of 2π steradians. Rewriting the above as an explicit integration over the two polar coordinates θ and ϕ, we have

$$I^{\uparrow}(\theta_r, \phi_r) = \int_0^{2\pi} \int_0^{\pi/2} \rho(\theta_i, \phi_i; \theta_r, \phi_r) I^{\downarrow}(\theta_i, \phi_i) \cos\theta_i \sin\theta_i \; d\theta_i d\phi_i \; . \tag{5.11}$$

As a special case, consider what happens if $\rho(\theta_i, \phi_i; \theta_r, \phi_r) = \rho_L$, where ρ_L is a constant. In that case, ρ_L can be taken out of the integral above, leaving

$$I^{\uparrow} = \rho_L \int_0^{2\pi} \int_0^{\pi/2} I^{\downarrow}(\theta_i, \phi_i) \cos\theta_i \sin\theta_i \; d\theta_i d\phi_i \; . \tag{5.12}$$

Note that the right hand side, and therefore $I^{\uparrow}$, no longer depends on (θ_r, ϕ_r), so we evidently have Lambertian reflection. Also, by comparing the remaining integral with (2.55), we realize that it represents the incident flux of radiation F_i, so that

$$I^{\uparrow} = \rho_L F_i \ . \tag{5.13}$$

If we multiply both sides of the above equation by π, we get an expression for the reflected flux (since $I^{\uparrow}$ is isotropic). From (5.2), we then conclude that

$$r = \pi \rho_L \ , \tag{5.14}$$

where r is the total (nondirectional) reflectivity of the Lambertian surface.

Let us now generalize to find the reflectivity $r(\hat{\boldsymbol{\Omega}}_i)$ corresponding to an arbitrary BDRF. For simplicity, we again consider the relationship between the reflected flux F_r and the incident flux $F_i = S_0 \cos\theta_i$ due to a columnated beam originating from a zenith angle θ_i:

$$F_r = r(\hat{\boldsymbol{\Omega}}_i) S_0 \cos\theta_i \ . \tag{5.15}$$

The reflected flux F_r can in turn be expressed in terms of an integral over the hemisphere of the upward reflected intensity $I^{\uparrow}$:

$$F_r = \int_{2\pi} I^{\uparrow}(\hat{\boldsymbol{\Omega}}) \cos\theta \, d\omega = \int_{2\pi} \rho(\hat{\boldsymbol{\Omega}}_i; \hat{\boldsymbol{\Omega}}_r) S_0 \cos\theta_i \, d\omega_r \ , \tag{5.16}$$

where we have substituted (5.9) into the integral. Setting (5.15) equal to (5.16) and dividing out the incident flux terms yields

$$r(\hat{\boldsymbol{\Omega}}) = \int_{2\pi} \rho(\hat{\boldsymbol{\Omega}}; \hat{\boldsymbol{\Omega}}_r) \, d\omega_r \ . \tag{5.17}$$

5.4 Applications to Meteorology, Climatology, and Remote Sensing

5.4.1 Solar Heating of Surfaces

The shortwave albedo of a surface has a large effect on the direct heating of the surface by sunlight and, ultimately, on the heating of air in contact with that surface. A field of freshly plowed bare soil (albedo $\sim$10%) will absorb almost 30% more solar radiation than a field of dry wheat (albedo ∼30%) and nine times as much as a layer of fresh dry snow (albedo ∼90%).

Whatever solar radiation is absorbed usually has the immediate effect of heating the surface. But much of that heat is eventually transferred to the atmosphere by way of (1) direct thermal conduction (*sensible heat flux*), (2) evaporation of surface moisture (*latent heat flux*), and / or (3) net longwave flux. Which one of these transfer mechanisms dominates depends on both atmospheric and surface conditions. A warm, humid atmosphere will tend to minimize the net loss of longwave radiation from the surface. A moist or vegetated surface will tend to transfer a proportionally larger fraction of its energy to the atmosphere in the form of latent heat.

If you have two dry, relatively bare land areas side by side with sharply differing albedos, then the darker surface, and therefore the air immediately above that surface, will be heated more rapidly by the absorption of solar radiation. If there is little wind, the sharp temperature difference that develops over time will trigger microscale circulations, as the warmer air over the darker surface rises in the form of a thermal updraft and is replaced by relatively cooler air flowing horizontally from the lighter surface. The latter air, of course, must be replaced by sinking air from above. Experienced glider pilots know how to exploit variations in land surface albedo so as to find the thermal updrafts that can help keep them aloft.

Problem 5.2: A field of snow that is just starting to melt is a special case, in that absorbed solar radiation will contribute primarily to a further phase change (solid to liquid) at a constant temperature of 0°C. Assume that the shortwave albedo of wet snow is 60% and that the solar flux reaching the surface on a particular sunny day is 500 W m^{-2}. The latent heat of fusion of ice L_f is 3.3×10^5 J / kg and you can assume that the density ρ_{ws} of wet snow stays constant (assuming all excess meltwater drains away) at around 200 kg m^{-3}. At what rate, in centimeters per hour, is the snow pack depleted by direct solar heating?