

Fig. 6.2: Blackbody emission curves at temperatures typical of the sun and of the earth and atmosphere. (a) The actual value of Planck's function, plotted on a logarithmic vertical axis. The diagonal dashed line corresponds to Wien's law (6.3). (b) Normalized depictions of the same functions as in (a), so that the areas under each curve are equal. Note that the vertical axis in this case is linear.

The physical dimensions of B_λ are thus those of intensity (power per unit area per unit solid angle) per unit wavelength; in common units: $\text{W m}^{-2}\mu\text{m}^{-1}\text{sr}^{-1}$.

Note that this interpretation is consistent with a point I made earlier: if you allow $d\lambda$ to go to zero, then the emitted radiation in the interval $[\lambda, \lambda + d\lambda]$ also goes to zero. Thus, a hypothetical (and unrealizable) detector that only could measure radiation at exactly one wavelength would detect no radiation at all. Consequently, any real detector used in atmospheric remote sensing has a finite

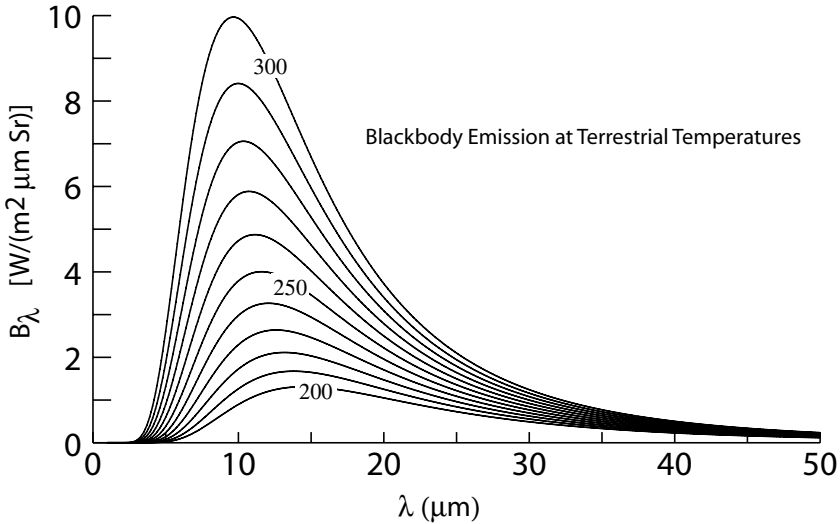


Fig. 6.3: The Planck (blackbody) function B_λ at temperatures typical of those found in the atmosphere.

spectral “bandwidth” that describes the range of wavelengths to which it responds. The greater the bandwidth, the more total radiant power the sensor receives and the greater its overall sensitivity, all other factors being equal.

Problem 6.1: Sometimes Planck’s function $B(T)$ may be expressed as a function of frequency ν or wavenumber $\tilde{\nu}$ instead of wavelength λ . Given that $B_\lambda(T) d\lambda$ must equal $B_\nu(T) d\nu$ when $d\nu$ and $d\lambda$ correspond to the same narrow interval of the spectrum, find the correct expression for B_ν as a function of ν only.

Examples of $B_\lambda(T)$ are shown for various temperatures in Figs. 6.2a and 6.3. Planck’s function is seen to have its peak at a wavelength that is inversely proportional to absolute temperature (see Wien’s Displacement Law, below). At any given wavelength, emission increases monotonically with increasing temperature. Emission is not symmetrically distributed about this peak; rather, the function drops off sharply at the short wavelength end of the spectrum while trailing off much more slowly toward long wavelengths.

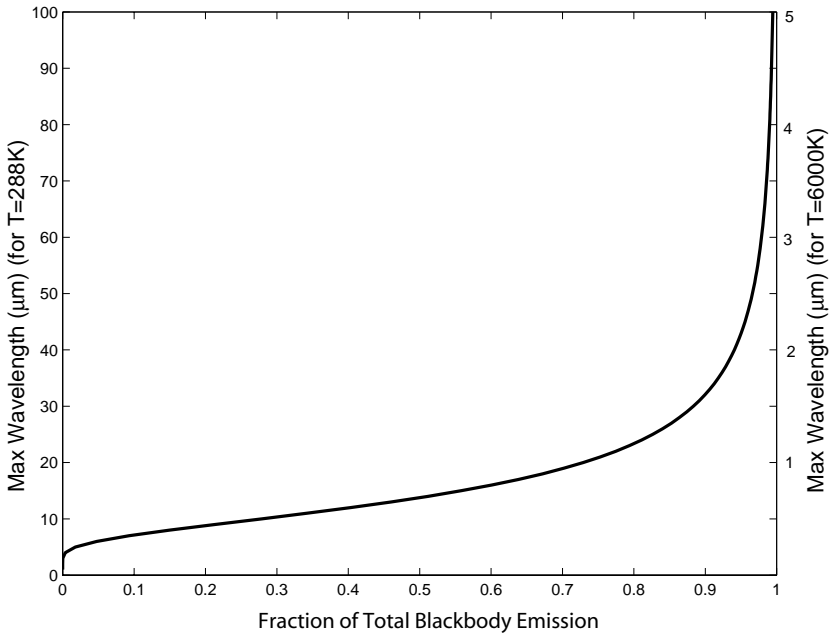


Fig. 6.4: The fraction of total blackbody emission contributed by wavelengths λ smaller than the threshold value indicated on the vertical axes. The horizontal axis gives the value of $\pi[\int_0^\lambda B_\lambda(T) d\lambda]/(\sigma T^4)$. The left axis corresponds to a blackbody with $T = 288$ K; the right axis corresponds to $T = 6000$ K. (Figure courtesy of S. Ackerman, with modifications)

6.1.2 Wien's Displacement Law

The wavelength λ_{\max} of the peak of the Planck function — i.e., that of maximum emission from a blackbody of temperature T is given by *Wien's Displacement Law*:

$$\lambda_{\max} = \frac{C}{T}, \quad (6.3)$$

where the constant $C = 2897 \mu\text{m K}$. Thus, peak emission from a blackbody with a temperature of 6000 K, similar to that of the sun, occurs at a wavelength of $\lambda_{\max} = 0.48 \mu\text{m}$, whereas typical atmospheric temperatures in the range 200K–300K yield peak emission in the range 9.6–14.4 μm .