Problem 6.7: Derive the Rayleigh-Jeans approximation (6.7) by expanding $\exp(hc/k_B\lambda T)$ in (6.1) as a power series and discarding quadratic and higher order terms. State what specific condition must be satisfied in order for the approximation to be valid. For the case that $T = 300$ K, determine the minimum wavelength for which the Rayleigh-Jeans approximation is valid to better than 1%.

6.2 Emissivity

Planck's function $B_\lambda(T)$ describes thermal emission from a blackbody which, as already noted, corresponds to the theoretical maximum possible emission from any real object. As such, it is an idealization. In calculations, one must therefore account for the degree to which real surfaces deviate from the ideal of a blackbody. We therefore introduce the concept of *emissivity*, which is nothing more than the ratio of what *is* emitted by a given surface to what *would be* emitted if it were a blackbody.

There are two cases of particular interest: 1) the emissivity at a single wavelength, and 2) emissivity over a broad range of wavelengths. The first case is most interesting in remote sensing applications, in which case we are primarily concerned with intensities, not fluxes. The second is generally of greatest concern for energy transfer calculations, in which case we care more about fluxes than intensities. The definitions that follow reflect these biases. You should be aware, however, that analogous, but slightly different definitions arise in other contexts.

6.2.1 Monochromatic Emissivity

Consider a surface that emits *less* radiation at a given wavelength λ and temperature T than that predicted by the Planck Function. If the actual intensity of the emission is I_λ, then the *monochromatic*

Table 6.1: Typical infrared emissivities (in percent) of various surfaces.

Water	92–96
Fresh, dry snow	82–99.5
Ice	96
Sand, dry	84–90
Soil, moist	95–98
Soil, dry plowed	90
Desert	90-91
Forest and shrubs	90
Skin, human	95
Concrete	71–88
Polished aluminum	1–5

emissivity of the surface is defined as

$$\boxed{\varepsilon_\lambda \equiv \frac{I_\lambda}{B_\lambda(T)}\ .} \tag{6.8}$$

Note that ε_λ might be a function of other variables, such as T, θ, and/or ϕ. In general $0 \le \varepsilon_\lambda \le 1$. When $\varepsilon_\lambda = 1$, the surface is effectively a blackbody at that wavelength.

6.2.2 Graybody Emissivity

By analogy to the monochromatic emissivity above, one may define a *graybody emissivity* ε as the ratio of the observed broadband flux F emitted by a surface to that predicted by the Stefan-Boltzmann relationship:

$$\boxed{\varepsilon \equiv \frac{F}{\sigma T^4}\ .} \tag{6.9}$$

Strictly speaking, no surface is truly “gray” over the full EM spectrum. Therefore, the use of a graybody emissivity (or absorptivity) in calculations invariably entails an approximation. It is nevertheless a convenient and useful simplification, especially for the types of problems encountered in an introductory survey of atmospheric radiation.

Return to: A First Course in Atmospheric Radiation

Sometimes, it is useful to apply the concept of graybody emissivity to a more limited range of wavelength $[\lambda_1, \lambda_2]$, in which case

$$\varepsilon(\lambda_1, \lambda_2) \equiv \frac{F(\lambda_1, \lambda_2)}{F_B(\lambda_1, \lambda_2)} , \tag{6.10}$$

where $F(\lambda_1, \lambda_2)$ is the actual flux emitted by the surface integrated between λ_1 and λ_2, and

$$F_B(\lambda_1, \lambda_2) \equiv \pi \int_{\lambda_1}^{\lambda_2} B_\lambda(T)\, d\lambda . \tag{6.11}$$

For example, one might reasonably attempt to characterize the emissivity of a surface in just the thermal IR band for the purpose of simple radiative balance calculations. Examples of IR emissivities are given in Table 6.1.

6.2.3 Kirchhoff's Law

I already alluded to the strong connection between how well a surface absorbs radiation at a given wavelength and how well it emits at the same wavelength. It's easy enough to confirm this experimentally by taking a light colored stone and a dark colored stone and heating them both in a furnace to the same high temperature. Turn out the lights and observe the red glow from each stone: the glow from the light stone will be much dimmer than that from the dark stone. In fact, if the stone were perfectly white (nonabsorbing), you would not be able to get it to glow at all, no matter how hot you heated it.

In quantitative terms, the relationship between absorptivity a and emissivity ε is embodied succinctly in *Kirchhoff's Law*, which states that

$$\boxed{\varepsilon_\lambda(\theta, \phi) = a_\lambda(\theta, \phi) .} \tag{6.12}$$

Note that this equivalence is strictly valid only for monochromatic radiation at a given wavelength λ and when the viewing directions θ and ϕ are specified, unless a (and therefore ε) are independent of these parameters over some range.