With the aid of the parameters given above, we can now begin to construct expressions for the upwelling and downwelling shortwave and longwave fluxes at the top of the atmosphere and between the atmosphere and the surface. Fig. 6.8 depicts the various components of these fluxes, and their physical interpretation is given here:

F_1	incident shortwave flux from the sun
F_2	transmitted portion of F_1
F_3	transmitted portion of F_4
F_4	shortwave flux reflected by the surface
F_5	longwave emission upward by atmosphere
F_6	longwave emission downward by atmosphere
F_7	transmitted portion of F_8
F_8	longwave emission by surface

Note that if we did not specify ahead of time that $\varepsilon = 1$, we would have also had to include terms that describe the upward reflection of F_6 from the surface.

We can write expressions for each of the flux terms using the parameters specified at the beginning of this section and the basic principles introduced earlier in this chapter. *Make sure you understand how each of the following expressions was obtained!*

$$F_1 = S \,, \tag{6.21}$$

$$F_2 = (1 - a_{\mathrm{sw}})F_1 = (1 - a_{\mathrm{sw}})S \,, \tag{6.22}$$

$$F_3 = (1 - a_{\mathrm{sw}})F_4 = A(1 - a_{\mathrm{sw}})^2 S \,, \tag{6.23}$$

$$F_4 = AF_2 = A(1 - a_{\mathrm{sw}})S \,. \tag{6.24}$$

In the following two expressions, we are invoking Kirchhoff's Law, which tells us that the longwave emissivity of the layer is the same as its absorptivity:

$$F_5 = a_{\mathrm{lw}}\sigma T_a^4 \,, \tag{6.25}$$

$$F_6 = F_5 = a_{\mathrm{lw}}\sigma T_a^4 \,, \tag{6.26}$$

$$F_7 = (1 - a_{\rm lw})F_8 = (1 - a_{\rm lw})\sigma T_s^4 \ , \tag{6.27}$$

$$F_8 = \varepsilon\sigma T_s^4 = \sigma T_s^4 \ . \tag{6.28}$$

The condition for radiative equilibrium is that the net fluxes (shortwave and longwave combined) at the top of the atmosphere and between the surface and the atmosphere are both zero. If this were not the case, then either the atmosphere or the surface, or both, would experience a net gain or loss of energy over time, leading to heating or cooling.

If we take the fluxes in the above table to be positive quantities, we thus have the following two conditions for radiative equilibrium:

$$F_{\rm net,top} = F_3 + F_5 + F_7 - F_1 = 0 \ , \tag{6.29}$$

$$F_{\rm net,sfc} = F_4 + F_8 - F_2 - F_6 = 0 \ . \tag{6.30}$$

Substituting our previous expressions for each flux term yields

$$A(1 - a_{\rm sw})^2 S + a_{\rm lw}\sigma T_a^4 + (1 - a_{\rm lw})\sigma T_s^4 - S = 0 \ , \tag{6.31}$$

$$A(1 - a_{\rm sw})S + \sigma T_s^4 - (1 - a_{\rm sw})S - a_{\rm lw}\sigma T_a^4 = 0 \ , \tag{6.32}$$

which we rearrange to give

$$(1 - a_{\rm lw})\sigma T_s^4 + a_{\rm lw}\sigma T_a^4 = S[1 - A(1 - a_{\rm sw})^2] \ , \tag{6.33}$$

$$\sigma T_s^4 - a_{\rm lw}\sigma T_a^4 = (1 - A)(1 - a_{\rm sw})S \ . \tag{6.34}$$

Note that if we define $x = \sigma T_s^4$ and $y = \sigma T_a^4$, then we have a pair of coupled linear equations in the two unknowns x and y. We can first solve these for x and y and then divide by σ and take the fourth root to get T_s and T_a:

$$T_s = \left\{ \frac{S}{\sigma} \left[1 - (1 - a_{\rm sw})A\right] \left(\frac{2 - a_{\rm sw}}{2 - a_{\rm lw}} \right) \right\}^{\frac{1}{4}} \ , \tag{6.35}$$

$$T_a = \left\{ \frac{S}{\sigma} \left[\frac{(1-A)(1-a_{\text{sw}})a_{\text{lw}} + [1+(1-a_{\text{sw}})A]a_{\text{sw}}}{(2-a_{\text{lw}})a_{\text{lw}}} \right] \right\}^{\frac{1}{4}} . \quad (6.36)$$

These expressions look complicated, but if we focus just on T_s and consider a couple of limiting cases, we can quickly get some useful insight into the role of the atmosphere in controlling global surface temperature.

The simplest case is when $a_{\text{lw}} = 0$ and $a_{\text{sw}} = 0$. This is equivalent to saying that there is no atmosphere at all, because it is completely invisible as far as both shortwave and longwave radiation is concerned. In fact, if you substitute these values and $S = S_0/4$ into (6.35) and simplify, you immediately recover (6.20), which is what you would expect.

Now consider the case that the surface is completely black (i.e., $A = 0$), while a_{lw} and a_{sw} may each be nonzero. We then have

$$T_s = \left[\frac{S_0}{4\sigma} \left(\frac{2-a_{\text{sw}}}{2-a_{\text{lw}}} \right) \right]^{\frac{1}{4}} . \quad (6.37)$$

This relationship is interesting, because it tells us that if $a_{\text{lw}} > a_{\text{sw}}$, then the surface temperature in our simple system will be warmer than would be the case without an atmosphere ($a_{\text{lw}} = a_{\text{sw}} = 0$) or, for that matter, any case where $a_{\text{lw}} = a_{\text{sw}}$.

In fact, our atmosphere is relatively transparent to shortwave radiation from the sun, while being comparatively absorbing at thermal IR wavelengths. We can approximate the earth's atmosphere by choosing $a_{\text{sw}} = 0.1$ and $a_{\text{lw}} = 0.8$. Thus, we expect the surface temperature of the earth to be substantially warmed by the absorption and re-emission of longwave radiation by the atmosphere. This warming effect due to the presence of an atmosphere is commonly known as the *Greenhouse effect*.

The above analysis assumes that the atmosphere either transmits or absorbs all of the solar radiation that is incident on it. It therefore cannot be used directly to obtain a realistic estimate of the equilibrium surface temperature of the earth. However, we can compensate for the loss of radiation due to reflection by clouds etc. by replacing S with $S(1-A_p)$, where $A_p = 0.30$ is the observed planetary albedo utilized earlier in our calculation from (6.20). We

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