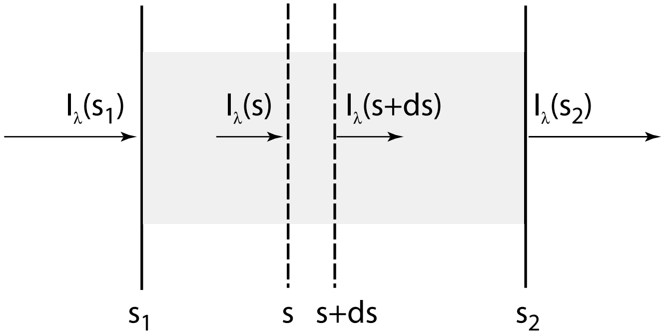

Fig. 7.3: Depletion of radiation over an infinitesimal path *ds* within an extinguishing medium.

a plume of exhaust from a diesel truck, the cloud-free atmosphere (based on the color of the setting sun).

7.2 Extinction Over a Finite Path

7.2.1 Fundamental Relationships

Let us now consider how to adapt (7.1) to a radiative path over which the extinction coefficient varies with location, since this is the situation we encounter in the atmosphere. We do not want to limit our attention to radiative paths aligned with the x-axis, so we generalize by replacing x with the geometric distance s along a ray in any arbitrary direction (Fig. 7.3). We further consider the attenuation of radiation over an infinitesimal path ds, which is chosen to be small enough so that (a) the extinction coefficient β_e is effectively constant within the interval, and (b) the incident radiation is attenuated by an infinitesimal amount dI_λ. We can then write

$$dI_\lambda \equiv I_\lambda(s+ds) - I_\lambda(s) = -I_\lambda(s)\beta_e(s)\ ds, \tag{7.4}$$

which we can rewrite as

$$\frac{dI_\lambda}{I_\lambda} \equiv d\log I_\lambda = -\beta_e\ ds. \tag{7.5}$$

Return to: A First Course in Atmospheric Radiation

In other words, the infinitesimal decrease in intensity dI_λ, expressed as a fraction of the incident intensity I_λ, is equal to the product of the local extinction coefficient times the infinitesimal path length ds.

In order to describe the extinction over an *extended* path between points s_1 and s_2, we simply integrate:

$$\log[I_\lambda(s_2)] - \log[I_\lambda(s_1)] = -\int_{s_1}^{s_2} \beta_e(s)\, ds, \tag{7.6}$$

or

$$\boxed{I_\lambda(s_2) = I_\lambda(s_1) \exp\left[-\int_{s_1}^{s_2} \beta_e(s)\, ds\right] .} \tag{7.7}$$

This equation gives a general form of *Beer's law*. From Beer's law follow several extremely important definitions and associated facts which I urge you to memorize before you continue reading:

- The integral quantity inside the brackets is called the *optical path* between points s_1 and s_2 (also known as *optical depth* or *optical thickness* when measured vertically in the atmosphere) :

$$\boxed{\tau(s_1, s_2) \equiv \int_{s_1}^{s_2} \beta_e(s)\, ds .} \tag{7.8}$$

  The optical path is dimensionless, as it must be, since it appears as the argument of the transcendental function exp(). It may take on any nonnegative value. It is zero when $s_1 = s_2$ or when $\beta_e = 0$ between s_1 and s_2; otherwise it is positive.

- By exponentiating the optical path τ, we get the *transmittance* between s_1 and s_2:

$$\boxed{t(s_1, s_2) \equiv e^{-\tau(s_1, s_2)} .} \tag{7.9}$$

  The transmittance is a dimensionless quantity ranging from near zero (for $\tau \rightarrow \infty$) to one (for $\tau = 0$). From (7.7) we see that

$$\boxed{I_\lambda(s_2) = t(s_1, s_2) I_\lambda(s_1) .} \tag{7.10}$$

Return to: A First Course in Atmospheric Radiation

Thus, $t \approx 1$ implies very weak attenuation of the beam between s_1 and s_2, whereas $t \approx 0$ implies near total extinction of the beam. Note that no matter how large τ, the transmittance is never identically zero, though it is often vanishingly small.

- If β_e happens to be constant between s_1 and s_2, then (7.9) simplifies to

$$\boxed{\tau = \beta_e(s_2 - s_1)\ .} \tag{7.11}$$

- Each (dimensionless) unit of optical path corresponds to a reduction of I_λ to $e^{-1} \approx 37\%$ of its original value.

- Consider the propagation of a ray along an extended path from s_1 to s_N. Break that path into several sub-paths; e.g., from s_1 to s_2, s_2 to s_3, s_{N-1} to s_N, etc. Then from (7.8) it can be readily shown that the total optical path can be written

$$\boxed{\tau(s_1, s_N) = \tau(s_1, s_2) + \tau(s_2, s_3) + \cdots + \tau(s_{N-1}, s_N)\ ,} \tag{7.12}$$

and the corresponding transmittance can be written

$$\boxed{t(s_1, s_N) = t(s_1, s_2) \cdot t(s_2, s_3) \cdot \cdots \cdot t(s_{N-1}, s_N)\ .} \tag{7.13}$$

In plain English, (1) *the total optical path equals the* **sum** *of the individual optical paths*; and (2) *the total transmittance equals the* **product** *of the individual transmittances.*

- Consider the propagation of a ray over an optical path $\tau(s_1, s_2) \ll 1$. That is, the medium is relatively transparent over the distance in question. This can occur if either that geometric distance Δs is sufficiently short or the extinction coefficient β_e is sufficiently small. In this case, the transmittance is approximated by

$$t = \exp(-\tau) \approx 1 - \tau(s_1, s_2) = 1 - \beta_e(s_2 - s_1), \tag{7.14}$$

where the equality on the right holds if we take β_e to be constant.

Return to: A First Course in Atmospheric Radiation