

where M is the mass of concentrated ink pigment added to the dish, H is the depth of the solution, and A is the horizontal area of the dish. Adding water increases H but does not change A or M ; therefore ρ decreases as the solution becomes more dilute.

The total vertical transmittance through the solution is thus given by

$$\begin{aligned} t &= \exp(-\tau) = \exp(-\beta_e H) \\ &= \exp \left[-k_e \left(\frac{M}{HA} \right) H \right] = \exp \left[-k_e \left(\frac{M}{A} \right) \right], \end{aligned} \quad (7.18)$$

so that we discover

$$\tau = k_e \frac{M}{A}. \quad (7.19)$$

As you can see, H does not appear on the right hand side. This is consistent with the observation that we can dilute the column as much or as little as we like and still get the same total transmittance, as long as M and A are fixed.

Note that k_e has dimensions of area per unit mass. You can think of this as an *extinction cross-section per unit mass*. The extinction cross-section can in turn be thought of as the equivalent area of an opaque object blocking the same total amount of radiation (assuming equal illumination everywhere). That is to say, if India ink pigment has a mass extinction coefficient k_e of, say, $100 \text{ m}^2 \text{ kg}^{-1}$, then one kg of pure ink pigment poured into a large body of clear water (and mixed thoroughly) has the capacity to block as much total sunlight from reaching the bottom as would a 100 m^2 sheet of opaque metal foil.² Obviously, the larger the area over which the ink is spread, the less intense the attenuation at any one point, but the total remains the same.

The concept of mass extinction coefficient is very handy precisely because, unlike the case for the volume extinction coefficient, it is nearly constant for most substances in the atmosphere irrespective of dilution, as long as all other variables (e.g., pressure, temperature, etc.) are held constant. This is particularly convenient when dealing with highly variable atmospheric constituents, such as water vapor.

²Strictly speaking, this equivalence holds only if the ink is spread out thinly enough (i.e., over a large enough area) so that the transmission is close to 100% at any point.

7.2.3 Extinction Cross-Section

There is one further extinction variable that frequently arises in contexts involving known concentrations (number densities) of particles. Such particles might be molecules of an absorbing gas, droplets of water in a cloud, or perhaps individual soot particles in a plume of smoke.

To return to our previous thought experiment, what if, instead of finding a proportionality between the volume extinction coefficient β_e and the mass density ρ , we instead relate the former to the *number density or concentration* N of microscopic particles of pigment in the ink? If we set up the proportionality as follows —

$$\beta_e = \sigma_e N, \quad (7.20)$$

we find that the constant of proportionality σ_e has dimensions of area. It is thus an extinction cross-section with the same physical interpretation as discussed earlier but now referenced to a single particle rather than a unit mass. It follows that

$$\sigma_e = k_e m, \quad (7.21)$$

where m is the mass per particle.³

One place where the idea of an extinction cross-section σ_e per particle is especially convenient and intuitive is in clouds. In the visible and infrared part of the spectrum, a single cloud droplet has an extinction cross-section that is similar, but not identical, to its geometric cross-section πr^2 , where r is the droplet radius. In fact, we may define an *extinction efficiency* Q_e according to

$$Q_e \equiv \frac{\sigma_e}{A}, \quad (7.22)$$

where A is the geometric cross-sectional area of the particle. In the case of a spherical droplet, $A = \pi r^2$, where r is the radius.

³Equations (7.20)–(7.21) assume that all of the particles are identical. We will generalize later to distributions of dissimilar particles.

One might surmise (incorrectly, as it turns out) that Q_e could range only from zero to one. That is, a particle should not be able to extinguish more radiation than its geometric cross-section would imply — in plain English again, it shouldn't cast a shadow bigger than itself. Unfortunately, however, our physical intuition lets us down in this instance. On the contrary, for cloud droplets in the visible spectrum, $Q_e \approx 2$ on average, and for certain wavelengths, it can be even larger! The reasons for this apparent paradox will be examined more closely in Chapter 12; for now, simply ascribe it to the wave properties of radiation.

7.2.4 Generalization to Scattering and Absorption

Throughout the last section, we have talked exclusively about *extinction*, and we defined a *mass extinction coefficient* and *extinction cross-section* to go along with the previously defined *volume extinction coefficient*. Recalling that extinction is the sum of absorption and scattering, it makes sense to define completely analogous mass- and particle-normalized quantities for those two processes:

$$\beta_a = \rho k_a = N\sigma_a \quad \beta_s = \rho k_s = N\sigma_s, \quad (7.23)$$

where k_a and k_s are the mass absorption coefficient and mass scattering coefficient, and σ_a and σ_s are the absorption cross-section and scattering cross-section. The latter can in turn be written in terms of absorption and scattering *efficiencies* as follows:

$$\sigma_a = Q_a A \quad \sigma_s = Q_s A, \quad (7.24)$$

where A is the geometric cross-sectional area of a particle.

It follows that the single scatter albedo can be written in terms of any of these quantities as follows:

$$\tilde{\omega} = \frac{\beta_s}{\beta_e} = \frac{k_s}{k_e} = \frac{\sigma_s}{\sigma_e}. \quad (7.25)$$