much more dramatically in the vertical than in the horizontal directions.

One obvious exception to the above rule is clouds. Many clouds *do* exhibit strong horizontal as well as vertical structure. Nevertheless, even clouds (especially *stratiform* clouds – e.g., stratus, stratocumulus, nimbostratus, altostratus, cirrostratus, etc.) are often organized in great sheets whose horizontal dimension greatly exceeds their vertical thickness. If you have ever spent time looking out a window during a cross-country airline flight, you may have noticed that you can sometimes traverse hundreds of kilometers over a more or less continuous sheet of clouds below you, especially in the vicinity of synoptic-scale weather disturbances. Thus, although it is by no means *always* the case, it is at least *sometimes* safe (and in any case very convenient) to treat even cloud layers as varying much more rapidly in the vertical than the horizontal.

For all of the above reasons, it is quite common (perhaps too common!) to treat the atmosphere for radiative purposes as *plane parallel*. That is, at a given location, we ignore horizontal variations in the structure of the atmosphere and assume instead that all relevant radiative properties depend strictly on the vertical coordinate z. Another term you will sometimes see that refers to the same approximation is *slab geometry*.

We also ignore the curvature of the earth in the plane parallel approximation, since any ray of light that is not traveling at a very shallow angle will pass through most of the mass of the atmosphere long before the earth's curvature comes into play. A good rule of thumb for the validity of this approximation is that $H/\cos\theta \ll R$, where θ is the angle of the ray relative to vertical, H is the effective depth of the atmosphere ($H \sim 10$ km for many purposes), and $R \approx 6380$ km is the radius of the earth. Obviously, this criterion is no longer satisfied if $\cos\theta$ becomes very small, as it does when θ approaches 90°. Usually, however, we are concerned primarily with radiation transporting energy more or less vertically through the atmosphere, in which case we can ignore this complication.

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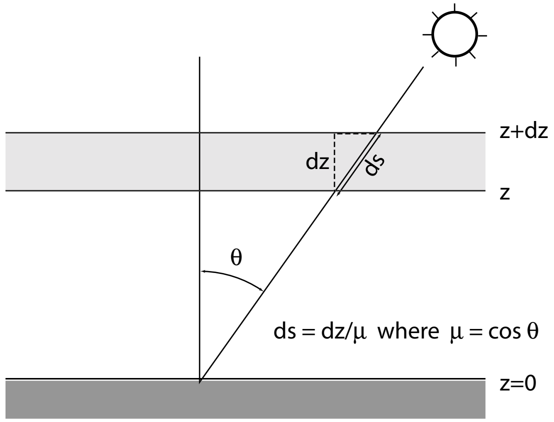

Fig. 7.4: Relationship between slant and vertical paths in a plane parallel atmosphere.

7.3.1 Definition

Mathematically, we invoke the plane parallel assumption via the following simplifications:

$$\beta_e(x, y, z) \approx \beta_e(z) , \tag{7.28}$$

$$T(x, y, z) \approx T(z) , \qquad \text{etc.} \tag{7.29}$$

Since everything now depends only on vertical distance z, the path distance s we used earlier in computing transmittance etc. along a ray (Fig. 7.4) may now be expressed as

$$\boxed{s = \frac{z}{\mu} ,} \tag{7.30}$$

where we have introduced the following new definition for convenience:

$$\boxed{\mu \equiv |\cos\theta| .} \tag{7.31}$$

As before, θ is the angle of propagation of the ray relative to zenith (i.e., straight up). Note, therefore, that $0 \leq \mu \leq 1$ regardless of whether the ray is propagating upward or downward.

Combining the above definitions with our earlier expressions for transmittance, optical path, etc., we have the following expressions valid for a plane parallel atmosphere:

The *optical thickness* between levels z_1 and z_2 is

$$\boxed{\tau(z_1, z_2) = \int_{z_1}^{z_2} \beta_e(z)\, dz\ ,} \tag{7.32}$$

and the transmittance for a ray propagating with direction μ is

$$\boxed{t(z_1, z_2) = \exp\left[-\frac{1}{\mu}\tau(z_1, z_2)\right]\ ,} \tag{7.33}$$

where $z_2 > z_1$.

Note that the optical thickness in a plane parallel atmosphere, as given above, is deliberately defined so as not to depend on the direction of propagation μ. Thus, it doesn't matter whether the sun is directly overhead ($\mu = 1$) or nearly setting on the horizon ($\mu \to 0$); the optical thickness of the atmosphere between levels z_1 and z_2 is the same. However, when computing the transmittance t of a ray of the sun as it passes between the two levels, it *is* of course necessary to account for μ, since the ray experiences a much longer optical *path* when the sun is low in the sky.

Just in case you've forgotten, let me reiterate that all of the above definitions still assume that we're talking about monochromatic radiation. That is, the parameters β_e, τ, t, etc., are all implicitly functions of wavelength.

Problem 7.5: At a certain wavelength in the visible band, the optical thickness of the cloud-free atmosphere is $\tau^* = 0.2$. Determine the transmittance of sunlight at this wavelength when the sun is 10° above the horizon.

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