Transmittance

From (7.37), the transmittance between altitude z and the top of the atmosphere (regardless of whether the radiation is propagating upward or downward) is determined by the optical depth $\tau(z)$ and by the cosine of the zenith (or nadir) angle $\mu = \cos\theta$:

$$t(z) = \exp\left[-\frac{\tau(z)}{\mu}\right] = \exp\left[-\frac{k_a w_1 \rho_0 H}{\mu} e^{-\frac{z}{H}}\right] . \tag{7.48}$$

This expression is admittedly a bit awkward-looking, in that one rarely encounters functions that are exponentials of exponentials! Apart from this minor peculiarity, it poses no computational problems.

Absorption

We already noted that, in a nonscattering atmosphere, the total absorption along a path is just equal to one minus the transmittance along that path. Thus, the total absorptance between z and the top of atmosphere is $a = 1 - t(z)$. A much more interesting and important question, however, is *where* that absorption is occurring.

Consider the absorption of solar radiation within the layer bounded by z_1 and z_2, where $z_2 > z_1$. This must be equal to the absorption between z_1 and the top of the atmosphere minus that occurring between z_2 and the top of the atmosphere:

$$a(z1, z2) = [1 - t(z_1)] - [1 - t(z_2)] = t(z_2) - t(z_1) . \tag{7.49}$$

Let us define $\Delta z = z_2 - z_1$ and look at the local *absorption per unit altitude*:

$$W(z) = \lim_{\Delta z \rightarrow 0}\left[\frac{a(z, z + \Delta z)}{\Delta z} = \frac{t(z + \Delta z) - t(z)}{\Delta z}\right] , \tag{7.50}$$

which of course reduces to

$$\boxed{W(z) = \frac{dt(z)}{dz} .} \tag{7.51}$$

The above relationship is extremely important, and you should spend some time thinking about its implications before you move on. It states that, for radiation incident at the top of the atmosphere, *the local rate of absorption within the atmosphere equals the local rate of change of transmittance from level z to the top of the atmosphere.*

Let us take things a step further: Noting that $t(z) = e^{-\tau(z)/\mu}$, we have

$$W(z) = \frac{d}{dz} e^{-\frac{\tau(z)}{\mu}} = -\frac{1}{\mu} e^{-\frac{\tau(z)}{\mu}} \frac{d\tau(z)}{dz} \, . \tag{7.52}$$

Noting further that $\tau(z) = \int_z^\infty \beta_e(z')dz'$, we have

$$\frac{d\tau(z)}{dz} = -\beta_e(z) \, , \tag{7.53}$$

thus

$$W(z) = \frac{\beta_e(z)}{\mu} e^{-\frac{\tau(z)}{\mu}} = \frac{\beta_e(z)}{\mu} t(z) \, . \tag{7.54}$$

That is, $W(z)$ equals the local extinction coefficient at level z times the transmittance from z to the top of the atmosphere.

Note that the above relationships (7.51) and (7.54) do *not* depend on any assumptions about the specific profile of absorption coefficient $\beta_a(z)$; that is, we haven't yet invoked the assumption of exponentially decaying density. In fact, they can be generalized as to apply to *any* path in *any* nonscattering, non-plane parallel atmosphere:

$$W(s) = \frac{dt(s,s')}{ds} = \beta_e(s) t(s,s') \, , \tag{7.55}$$

where s is the distance along the path toward the source of the radiation at $s = s'$ and $t(s)$ is the transmittance between s and s'.

Does this relationship between local absorption and the rate of change of transmittance make physical sense? Consider a perfectly transparent atmosphere. The transmittance $t(z)$ is then constant and equal to one for all z; therefore $dt/dz = 0$, and the local absorption per unit distance is also zero, as you would expect. Now consider the rate of absorption deep within a strongly absorbing atmosphere. Except near the very top of that atmosphere, $t(z) \approx 0$ for all z, so again $dt/dz \approx 0$ and there is no local absorption of radiation incident at the top of the atmosphere. That is to say, there's essentially nothing left *to* absorb, because everything already got absorbed at a higher altitude!

To summarize, it is *by definition* the range of z over which the transmittance drops from near one to near zero (when moving downward through the atmosphere) that most of the incident radiation is absorbed, and it is of course where the change in transmittance is most rapid that the absorption is most rapid.[7]

Let us conclude this discussion by working out the math for our idealized exponential atmosphere. All that is necessary is to substitute our expressions for $\beta_a(z)$ and $t(z)$, from (7.42) and (7.48) respectively, into (7.54), yielding

$$W(z) = \frac{1}{\mu} k_a w_1 \rho_0 e^{-\frac{z}{H}} \exp\left[-\frac{k_a w_1 \rho_0 H}{\mu} e^{-\frac{z}{H}} \right] , \qquad (7.56)$$

which can be written in simpler form by substituting (7.45):

$$W(z) = \frac{\tau^*}{H\mu} e^{-\frac{z}{H}} \exp\left[-\frac{\tau^*}{\mu} e^{-\frac{z}{H}} \right] . \qquad (7.57)$$

The characteristic shape of $W(z)$ is depicted schematically by the dotted line in Fig. 7.10. As explained above, it is nearly zero at high altitudes where the atmospheric density is vanishingly small; it is also zero at low levels, below the maximum depth to which radiation can penetrate. Somewhere in between, of course, it has its maximum value, which is where $t(z)$ is changing most rapidly.

Qualitatively, we expect the *altitude of peak absorption* to depend on the strength of absorption; i.e., on the value of k_e at the wavelength in question. If k_e is small, the atmosphere is relatively transparent, and radiation penetrates to lower altitudes, or even to ground level, before the rate of absorption becomes strong. If k_e is large, then absorption is strongest at high altitudes where the density is still very small; at lower levels no radiation remains to be absorbed.

We can find the altitude of the peak of $W(z)$ using the standard approach: Take the derivative with respect to z, set the result equal to zero, and solve for z:

$$\frac{dW(z)}{dz} = \frac{d}{dz} \frac{\tau^*}{H\mu} e^{-\frac{z}{H}} \exp\left[-\frac{\tau^*}{\mu} e^{-\frac{z}{H}} \right] , = 0 \qquad (7.58)$$

[7]The layer of the atmosphere in which solar radiation of a particular wavelength and incidence angle is absorbed — i.e., where $W(z) > 0$ — is sometimes referred to as the *Chapman layer*, after the scientist who first studied it.

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