together, in which case we refer to the *total transmittance* as

$$\boxed{t = t_{\mathrm{dir}} + t_{\mathrm{diff}}\ ,} \tag{7.64}$$

so that

$$\boxed{t + r + a = 1\ .} \tag{7.65}$$

The above properties of clouds are of profound importance for the atmospheric radiation budget. In particular, the reflectivity r of a cloud layer helps determine how much of the solar radiation incident on the top of the atmosphere gets immediately reflected back to space, forever lost as far as the energy budget of the earth and atmosphere is concerned. The total transmittance t limits the amount of solar radiation available to directly heat the surface. And of course the in-cloud absorptance a defines the fraction that contributes to direct heating of the atmospheric layer in which the cloud resides.

For any given cloud, the values of the above four variables depend on its optical thickness τ^*, the single scatter albedo $\tilde{\omega}$, and details of *how* radiation is scattered by the constituent cloud droplets. The three properties t_{diff}, r, and a all include contributions from *multiple scattering* of radiation within the cloud layer. We will not have the tools to address multiple scattering until a later chapter.

We do, however, already have the tools to evaluate the optical thickness τ^* of a cloud layer in terms of the cloud droplet radii r, concentrations N, and extinction efficiencies Q_e. The direct transmittance t_{dir} then follows from (7.33):

$$\boxed{t_{\mathrm{dir}} = e^{-\tau^*/\mu}\ .} \tag{7.66}$$

Monodisperse Cloud

Let us start by considering an ideal plane parallel cloud composed entirely of cloud droplets of identical radius r in a concentration N per unit volume. Such a cloud composed of equal-sized droplets is called *monodisperse* , as contrasted with a more realistic cloud

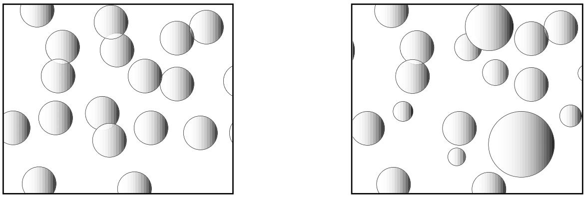

Fig. 7.11: Illustration of the definitions of monodisperse and polydisperse for cloud water droplets.

composed of *polydisperse* cloud droplets (Fig. 7.11). From (7.20) and (7.22), we have the volume extinction coefficient

$$\beta_e = N Q_e \pi r^2 \ . \tag{7.67}$$

Usually, it is difficult to directly measure either N or r but somewhat easier to measure or estimate the cloud water density ρ_c, which is the mass of condensed cloud water per unit volume of air. Typical values for ρ_c lie in the range 0.1–1 g / m^3, where the lower values are usually found in stratiform clouds experiencing weak uplift, whereas values in excess of 1 g / m^3 are most often found in the cores of vigorous convective updrafts.

For our monodisperse cloud, the cloud water density is just the number concentration of droplets times the mass of water in each droplet:

$$\rho_c = N \frac{4}{3} \pi r^3 \rho_l \ , \tag{7.68}$$

where $\rho_l \approx 1000$ kg / m^3 is the density of pure water. Combining with (7.16) and (7.67), we find that the volume extinction index-Cloud!extinction coefficient indexExtinction coefficient!in cloud coefficient is given by

$$\beta_e = N Q_e \pi r^2 = k_e \rho_c = k_e N \frac{4}{3} \pi r^3 \rho_l \ , \tag{7.69}$$

which allows us to solve for the mass extinction coefficient:

$$k_e = \frac{3 Q_e}{4 \rho_l r} \ . \tag{7.70}$$

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This result is interesting, but perhaps not surprising. It tells us that the same mass of water broken up into droplets extinguishes more radiation if the droplets are small and numerous than if they are large and relatively few.

Is (7.70) consistent with everyday experience? Consider moderate falling rain and thick fog. At visible wavelengths, $Q_e \approx 2$ for both cases. But for rain, the drop radius r is on the order of 1 mm; for fog it is a hundred times smaller, or about 10 μm. Substituting these values into (7.70) yields $k_e \approx 1.5\ \mathrm{m^2/kg}$ for rain and $k_e \approx 150\ \mathrm{m^2/kg}$ for fog. Yet both rain and fog can yield liquid water concentrations ρ_c on the order of 0.1 g per cubic meter of air. Thus, the volume extinction coefficient β_e can be on the order of 0.15 km^{-1} and 15 km^{-1} for rain and fog. In the first case, the transmittance along a 1 km path is 86%; in the second, it is for all practical purposes zero!

Let's return to the problem of determining the optical thickness of the cloud layer with cloud water density profile $\rho_c(z)$ between cloud base z_{bot} and cloud top z_{top}:

$$\tau^* = \int_{z_{\mathrm{bot}}}^{z_{\mathrm{top}}} \beta_e(z)\, dz = \int_{z_{\mathrm{bot}}}^{z_{\mathrm{top}}} k_e \rho_c(z)\, dz\ . \tag{7.71}$$

Since we're assuming that k_e is constant in this problem, we can take it out of the integral and simply write

$$\tau^* = k_e L\ , \tag{7.72}$$

where the *liquid water path* (vertically integrated mass of cloud water per unit horizontal area) is defined as

$$L \equiv \int_{z_{\mathrm{bot}}}^{z_{\mathrm{top}}} \rho_c(z)\, dz\ . \tag{7.73}$$

Combining (7.72) and (7.70) and using $Q_e \approx 2$, we get

$$\tau^* \approx \frac{3L}{2\rho_l r}\ . \tag{7.74}$$

To summarize, the total optical depth of our plane parallel cloud layer is proportional to the liquid water path L and inversely proportional to radius of the constituent cloud droplets.[8]

[8]We are neglecting any contribution by the air itself to any absorption within the cloud layer. At some wavelengths in the solar band, absorption by water vapor or other constituents cannot be neglected.

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