Problem 7.11: A certain cloud layer has geometric thickness $H = 0.1$ km and liquid water path $L = 0.01$ kg m^{-2}. Taking $Q_e \approx 2$ and the solar zenith angle $\theta = 60^\circ$, compute the direct transmittance t_{dir} for (a) $N = 100$ cm^{-3} (characteristic of clean maritime environments), and (b) $N = 1000$ cm^{-3} (characteristic of continental environments).

Polydisperse Cloud†

In our above analysis of the effect of droplet concentration and radius on cloud optical thickness, we assumed that all cloud droplets had the same size. This is a reasonable approach when the purpose is just to gain qualitative insight into how the radiative properties of clouds depend on drop size and number. It is insufficient, however, for anything that requires accurate calculations of those properties.

In reality, of course, the droplets found in any given cloud are distributed over a range of sizes. We may describe the *drop size distribution* in any given case via a function $n(r)$, such that

$$n(r)\,dr = \left\{ \begin{array}{l} \text{number of droplets (per unit volume of} \\ \text{air) whose radii fall in the range } [r, r+dr] \end{array} \right\} . \tag{7.77}$$

It follows that $n(r)$ has dimensions of "per unit volume per unit interval of r", or $length^{-4}$. Often, the units will be written as [m^{-3} μm^{-1}].

With the above definition of $n(r)$, one may immediately derive several related quantities. For example, the *total* number of droplets of all sizes (per unit volume), is just

$$N = \int_0^\infty n(r)\,dr . \tag{7.78}$$

The total number of droplets whose radius is smaller than some radius r' is

$$N(r < r') = \int_0^{r'} n(r)\,dr . \tag{7.79}$$

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The total surface area (per unit volume of air) contributed by droplets of all sizes is obtained by first multiplying $n(r)$ by the expression for the surface area of a single droplet of radius r and then integrating over all sizes:

$$A_{\mathrm{sfc}} = \int_0^\infty n(r) \left[4\pi r^2\right] \, dr \,. \tag{7.80}$$

An analogous approach can be used to obtain those radiative and cloud physical quantities that we already worked with in the case of a monodisperse cloud. For example, the local cloud water density is given by

$$\rho_c = \int_0^\infty n(r) \left[\rho_l \frac{4\pi}{3} r^3\right] \, dr \,. \tag{7.81}$$

Finally, the local volume extinction coefficient is obtained as

$$\beta_{\mathrm{e}} = \int_0^\infty n(r) \left[Q_{\mathrm{e}}(r) \pi r^2\right] \, dr \,, \tag{7.82}$$

where the expression in brackets represents the extinction cross-section of a single droplet of radius r. The mass extinction coefficient can then be written as

$$k_{\mathrm{e}} \equiv \frac{\beta_{\mathrm{e}}}{\rho_c} = \frac{\int_0^\infty n(r) \left[Q_{\mathrm{e}}(r) \pi r^2\right] \, dr}{\int_0^\infty n(r) \left[\rho_l \frac{4\pi}{3} r^3\right] \, dr} \,. \tag{7.83}$$

If we again take $Q_{\mathrm{e}} \approx 2$ for all r (valid when the droplets are large compared with the wavelength of the radiation), we find that the above simplifies to

$$k_{\mathrm{e}} \approx \frac{3}{2\rho_l r_{\mathrm{eff}}} \,, \tag{7.84}$$

where the *effective drop size* r_{eff} is *defined* as

$$r_{\mathrm{eff}} \equiv \frac{\int_0^\infty n(r) r^3 \, dr}{\int_0^\infty n(r) r^2 \, dr} \,. \tag{7.85}$$

The total optical depth of the cloud layer is then approximately

$$\tau^* \approx \frac{3L}{2\rho_l r_{\mathrm{eff}}} \,, \tag{7.86}$$

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