come in two basic flavors: *wide-band emission* models and *narrow-band transmission* models. The former represent the most radical simplification of the radiative transfer equation and are widely used in applications (e.g., climate models) in which computational efficiency is of paramount importance. Narrow-band transmission models, which we will discuss in this book, are not as efficient but are capable of yielding more accurate results while still being substantially faster to execute than LBL codes.

The $k$-distribution method is a relatively recent innovation and is rapidly gaining favor in view of its ability to achieve fairly accurate results with two or three orders of magnitude less computer time than LBL methods.

For both narrow-band transmission models and the $k$-distribution method, the longwave spectrum is subdivided into $N$ intervals $\Delta\tilde{\nu}_i$ that are each

1. large enough to encompass a significant number of absorption lines associated with a particular atmospheric constituent, and yet

2. small enough so that the Planck function $B_{\tilde{\nu}}(T)$ can be considered approximately constant and equal to $\overline{B}_i$ over the range.

Thus, the downwelling broadband longwave flux at level $z$ can be written

$$F^{\downarrow}(z) = \int_0^{\infty} F^{\downarrow}_{\tilde{\nu}}(z)\, d\tilde{\nu} = \sum_{i=1}^{N} \int_{\Delta\tilde{\nu}_i} F^{\downarrow}_{\tilde{\nu}}(z)\, d\tilde{\nu} = \sum_{i=1}^{N} F^{\downarrow}_i(z)\ , \tag{10.11}$$

where the flux contribution from each spectral interval can be expanded as

$$F^{\downarrow}_i(z) = \int_{\Delta\tilde{\nu}_i} F^{\downarrow}_{\tilde{\nu}}(z)\, d\tilde{\nu} = \int_{\Delta\tilde{\nu}_i} \int_z^{\infty} \pi B_{\tilde{\nu}}[T(z')] \frac{\partial t_{\tilde{\nu}}(z', z; \overline{\mu})}{\partial z'}\, dz'\ . \tag{10.12}$$

Making use of condition 2 above, and reordering the differentiation and integration operators, we can write

$$F^{\downarrow}_i(z) = \pi\Delta\tilde{\nu}_i \int_z^{\infty} \overline{B}_i[T(z')] \frac{\partial \mathcal{T}_i(z', z; \overline{\mu})}{\partial z'}\, dz'\ , \tag{10.13}$$

where the *band-averaged transmittance* is

$$\mathcal{T}_i(z', z; \overline{\mu}) = \frac{1}{\Delta\tilde{\nu}_i} \int_{\Delta\tilde{\nu}_i} t_{\tilde{\nu}}(z', z; \overline{\mu})\, d\tilde{\nu} = \frac{1}{\Delta\tilde{\nu}_i} \int_{\Delta\tilde{\nu}_i} e^{-\tau_{\tilde{\nu}}(z', z)/\overline{\mu}}\, d\tilde{\nu} \,. \tag{10.14}$$

Analogous equations can of course be found for the upwelling flux $F_i^{\uparrow}(z)$.

The efficient computation of broadband fluxes evidently boils down to the problem of finding good approximations to $\mathcal{T}_i$ between two levels for the particular spectral interval $\Delta\tilde{\nu}_i$. In practice, this problem is solved in two steps:

1. Develop a method for efficiently estimating $\mathcal{T}_i$ over an arbitrary *homogeneous* path — that is, one over which line shapes and strengths can be considered constant, and

2. generalize the above method to inhomogeneous (e.g. vertical) paths, over which linewidths can be expected to vary substantially owing to pressure broadening, etc.

Band transmission models and the *k*-distribution method offer two rather different frameworks for tackling both steps. We will consider each in turn.

## 10.2 Band Transmission Models

Previously, when we examined the extinction of radiation (Chapter 7), we were always concerned with the fate of *monochromatic* radiation along a finite path between points $s_1$ and $s_2$. The transmittance $t(s_1, s_2)$ in this case is very nicely described by Beer's Law (7.7), which is both easy to remember and easy to apply. In particular, you'll remember that one consequence of Beer's Law is that the transmittance over an extended path is equal to the product of the transmittances over a series of shorter paths.

You can forget about Beer's law in the case of spectrally averaged transmission, except in the rare case that the medium happens to be gray over the relevant interval $\Delta\tilde{\nu}$ – that is, when $\tau_{\tilde{\nu}}$ is independent of $\tilde{\nu}$. Qualitatively, this is because radiation with different wavenumbers within the band are depleted at different rates.

At some wavenumbers for which the medium is transparent, there is very little depletion even over long paths, and these wavenumbers ensure that the average band transmittance remains nonzero even at a great distance from the source. At other wavenumbers (e.g., at line centers), radiation is depleted rapidly, contributing to a sharp decrease in band transmission over a short distance. But once those wavenumbers are gone, they can't be depleted further, and so the transmission ceases to decrease as rapidly with further distance along the beam.

**Problem 10.1:** Imagine that between wavenumbers $\tilde{\nu}_1$ and $\tilde{\nu}_2$, the atmosphere is perfectly transparent, whereas between $\tilde{\nu}_2$ and $\tilde{\nu}_3$, the absorption coefficient $\beta_a$ is constant and nonzero.

(a) Find an expression for the average transmittance $\mathcal{T}$ over a pathlength $s$ for the spectral band $\Delta\tilde{\nu} = \tilde{\nu}_3 - \tilde{\nu}_1$.

(b) Sketch a graph of $\mathcal{T}(s)$ for the case that $\tilde{\nu}_2$ is the midpoint of the interval, being sure to show the asymptotic behavior of $\mathcal{T}$ for large $s$.

(c) Assuming that the radiation incident at point $s$=0 is "white", compute the fraction that is absorbed by the time it reaches $s = 1$ km, if $\beta_a = 3\ \mathrm{km}^{-1}$.

(d) Assume that the radiation that survives the transit in (c) goes on to traverse the path from $s = 1$ km to $s = 2$ km. What fraction of the radiation that begins this second leg is absorbed by the time it reaches the end?

(e) Explain why the fraction absorbed over the first kilometer is substantially different than the fraction absorbed over the second kilometer.

We will now consider the problem of band transmission in a more quantitative fashion. To start with, it is instructive to consider the band-averaged transmission and absorption properties in the idealized case of a single isolated absorption line. We will use this example to gain some basic insight and to introduce some definitions that can later be applied to more complex cases.

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