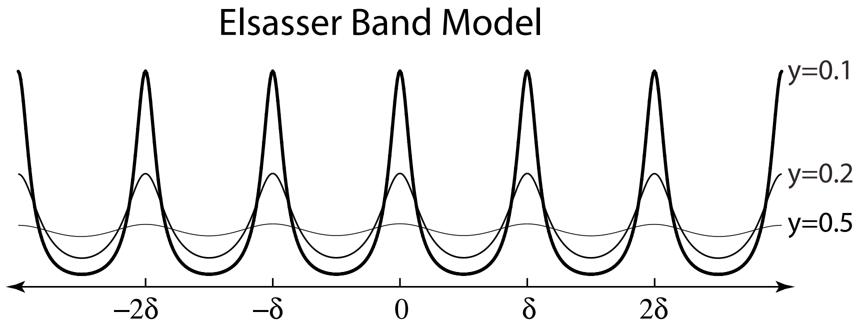

Fig. 10.2: Schematic depiction of the absorption coefficient k in the Elsasser (regular) band model, for three different values of the grayness parameter $y \equiv \alpha_L / \delta$.

With substitution of the Lorentz line shape, we have

$$k(\tilde{\nu}) = \sum_{n=-\infty}^{n=+\infty} \frac{S}{\pi} \frac{\alpha_L}{[(\tilde{\nu} - n\delta)^2 + \alpha_L^2]} \; . \tag{10.38}$$

Elsasser showed that this is mathematically equivalent to

$$k(\tilde{\nu}) = \frac{S}{\delta} \frac{\sinh(2\pi y)}{\cosh(2\pi y) - \cos(2\pi x)} \; , \tag{10.39}$$

where

$$y \equiv \frac{\alpha_L}{\delta}, \qquad x \equiv \frac{\tilde{\nu}}{\delta} \; . \tag{10.40}$$

y can be regarded as a "grayness parameter": if y is large, then adjacent lines strongly overlap and blur together, so that the line structure is increasingly obscured; for small y, the lines are well separated and resemble isolated lines for small to moderate mass paths (Fig. 10.2).

Because the pattern of lines is periodic, the band transmission $\mathcal{T}$ over a large spectral range can be obtained by integrating the monochromatic transmission over just a single interval δ, which is equivalent to one unit of the nondimensional wavenumber parameter x:

$$\mathcal{T} = \int_{-1/2}^{1/2} \exp[-k(x)u] \; dx \; , \tag{10.41}$$

or

$$\mathcal{T} = \int_{-1/2}^{1/2} \exp\left[-\frac{2\pi\tilde{u}y\sinh(2\pi y)}{\cosh(2\pi y) - \cos(2\pi x)}\right] \; dx \; , \tag{10.42}$$

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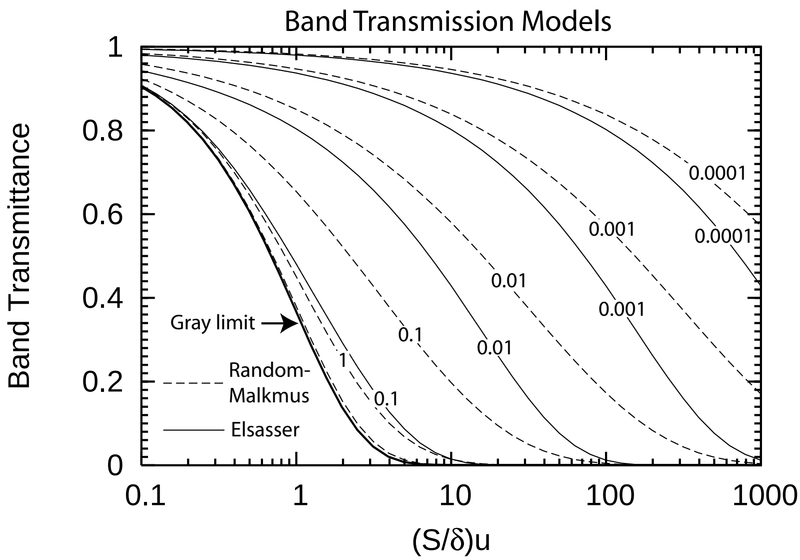

Fig. 10.3: Comparison of the Elsasser (solid curves) and random-Malkmus (dashed curves) band models for varying values of the grayness parameter y (labels on curves). For both models, curves approach the gray limit (Beer's Law) when $y \gg 1$.

where the nondimensional mass path $\tilde{u}$ is as defined in (10.29).

Unfortunately, the above integral cannot be solved in closed form. However, there are important limiting cases. The first is when y is large (order 10 or greater), in which case the line width is much greater than the spacing between lines. In this case, the medium is effectively gray, and the band transmission reduces to

$$\mathcal{T} = \exp(-2\pi y\tilde{u}) = \exp(-Su/\delta) \ . \tag{10.43}$$

In other words, we're back to Beer's law, with mass absorption coefficient $k = S/\delta$.

The second limiting case of interest is for $\tilde{u} \gg 1$. The band transmission then has the asymptotic form

$$\mathcal{T} \approx 1 - \text{erf}\left[\pi y\sqrt{2\tilde{u}}\right] \ , \tag{10.44}$$

where $\text{erf}(x)$ is the so-called *error function*, defined as

$$\text{erf}(x) \equiv \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} \, dt \ . \tag{10.45}$$

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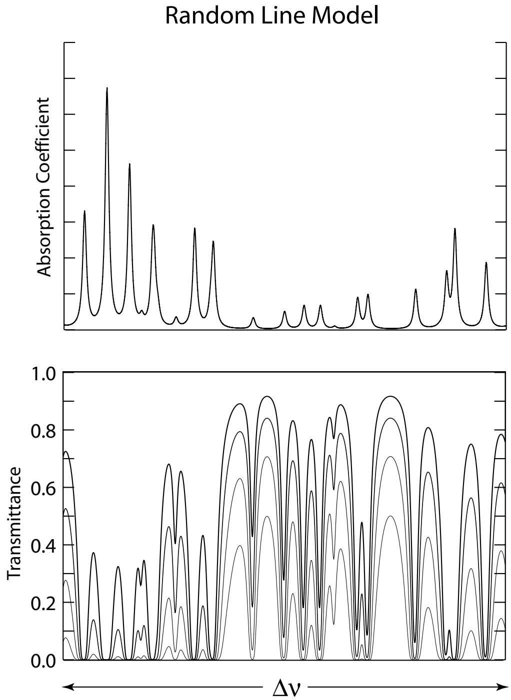

Fig. 10.4: Example of the distribution of line locations and strengths according to a random / Malkmus model. Upper panel shows the absorption coefficient (arbitrary units); lower panel depicts the spectrum of transmittance for four different mass paths.

The behavior of Elsasser band transmission $\mathcal{T}(u)$ is depicted as solid curves in Fig. 10.3 for various values of y. The main thing to take away from this plot is that when the spacing between lines is substantial ($y \ll 1$), the transmission decreases in a sub-exponential fashion.

10.2.4 The Random/Malkmus Band Model

Line spectra for important nonlinear molecules like water vapor, ozone, methane, etc. do not exhibit any of the regularity assumed by the Elsasser model. Even the P- and R-branches of a linear molecule like CO_2 band is more complicated in reality than might be expected based on our simplified discussion in Chapter 9. Consequently, the