section determines the value of g that corresponds to our chosen k. Now follow the vertical dotted line down to where it intersects the curve in Fig. 10.5c. This intersection determines a new value of k *at the higher pressure level* for which $g(k)$ has the same value as for our original k at low pressure. Now follow the horizontal dotted line back to the left and note where it crosses $k(\tilde{\nu})$. These intersections define the new set of frequencies at high pressure that were determined by our particular choice of k at low pressure. For the most part, these frequencies are very close to those determined by the intersections in Fig. 10.5a. The correspondence is not perfect — for example, the fifth line from the left has two intersections in the top panel that have no counterparts in the lower panel.

More generally, we can determine the complete mapping between k at the two pressure levels, based on the requirement of equal values of g. The results for this example are shown in Fig. 10.5f. Using this mapping, we can test how well we are able to predict $k(\tilde{\nu})$ at one pressure based on $k(\tilde{\nu})$ at the other pressure. Fig. 10.5g depicts both the original $k(\tilde{\nu})$ at low pressure (solid curve) and the predicted $k(\tilde{\nu})$, based on applying the mapping in panel (f) to the high pressure $k(\tilde{\nu})$ in panel (c). Over most of the spectral interval, the agreement is nearly perfect. Only in the vicinity of a few line centers (e.g., that fifth line from the left) does the agreement break down noticably. However, these isolated errors occupy such a small fraction of the spectral band that they don't introduce much error into the calculation of $\mathcal{T}(u)$.

The bottom line is that the correlated-k method, while not perfect, typically allows fluxes and heating rates to be calculated with overall errors of less than 1%. Most importantly, it does so with at least three orders of magnitude less computational effort than a brute-force LBL calculation. If you ever have occasion to work with a modern model for computing band-averaged IR radiances or fluxes in the atmosphere, there's a good chance that you can find a correlated-k procedure lurking somewhere inside.

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10.4 Applications to Meteorology, Climatology, and Remote Sensing

The previous section outlined the methods used to compute broadband upwelling and downwelling fluxes in a cloud-free atmosphere. Of course hardly anyone computes fluxes just for the fun of it; rather, they do it because it is an essential part of modeling the energy budget of the atmosphere.

10.4.1 Fluxes and Radiative Heating/Cooling

Radiative Heating Equations

In Section 2.7, we defined the net flux as

$$F^{\text{net}}(z) \equiv F^{\uparrow}(z) - F^{\downarrow}(z) \; . \tag{10.53}$$

Depending on the context, we might be interested in the net flux computed (or measured) for all wavelengths, for just the shortwave or longwave band, or even for a narrow spectral interval. In any of these cases, F^{net} represents the corresponding net upward flow of radiative energy, measured in watts per meter squared, through a unit horizontal area at level z.

Now consider a thin layer of the atmosphere with its base at altitude z and its upper boundary at $z + \Delta z$. The net flux $F^{\text{net}}(z)$ represents the rate at which radiative energy enters the bottom of this layer; likewise, $F^{\text{net}}(z + \Delta z)$ gives the rate at which energy leaves at the top of the layer. If the two fluxes are equal, then there is no net change over time in the internal energy of the layer. If they are not equal, then the layer must be experiencing a net gain or loss of energy.

It follows that the *radiative heating rate* at level z is given by

$$\boxed{\mathcal{H} \equiv -\frac{1}{\rho(z) C_p} \frac{\partial F^{\text{net}}}{\partial z}(z) \; ,} \tag{10.54}$$

where $\rho(z)$ is the air density at level z and $C_p = 1005\,\text{J}/(\text{kg K})$ is the specific heat capacity of air at constant pressure. The minus sign is needed because an *increase* in $F^{\rm net}$ with height implies a net *loss* of energy from level z. Traditionally, $\mathcal{H}$ is expressed in units of °C / day. When the value of $\mathcal{H}$ is negative (as it is more often than not), one might prefer to speak instead of positive *radiative cooling rate*.

In order to utilize the band model machinery we developed earlier, let's confine our attention to the heating / cooling rate associated with a particular spectral interval $\Delta\tilde{\nu}_i$. The complete expressions for upwelling and downwelling flux, including boundary contributions, are then

$$F_i^{\uparrow}(z) = F_i^{\uparrow}(0)\mathcal{T}(0,z) + \Delta\tilde{\nu}_i \int_0^z \pi\overline{B}_i(z')\frac{\partial\mathcal{T}_i(z',z)}{\partial z'}\,dz' \,, \qquad (10.55)$$

$$F_i^{\downarrow}(z) = F_i^{\downarrow}(\infty)\mathcal{T}(z,\infty) - \Delta\tilde{\nu}_i \int_z^\infty \pi\overline{B}_i(z')\frac{\partial\mathcal{T}_i(z,z')}{\partial z'}\,dz' \,, \qquad (10.56)$$

where, as usual, $\mathcal{T}(z,z')$ is the band-averaged flux transmittance between levels z and z'. Note that we are using $\overline{B}_i(z)$ as a shorthand notation for the average value of $B[T(z)]$ in the ith spectral interval.

Let's use the above expressions to evaluate $F^{\rm net}$ and, from there, $\partial F^{\rm net} / \partial z$, as required for the heating rate in (10.54):

$$\begin{aligned} F_i^{\rm net}(z) = {} & F_i^{\uparrow}(0)\mathcal{T}(0,z) - F_i^{\downarrow}(\infty)\mathcal{T}(z,\infty) \\ & + \Delta\tilde{\nu}_i \int_0^z \pi\overline{B}_i(z')\frac{\partial\mathcal{T}_i(z',z)}{\partial z'}\,dz' \\ & + \Delta\tilde{\nu}_i \int_z^\infty \pi\overline{B}_i(z')\frac{\partial\mathcal{T}_i(z,z')}{\partial z'}\,dz' \,, \end{aligned} \qquad (10.57)$$

$$\begin{aligned} \frac{\partial F_i^{\rm net}(z)}{\partial z} = {} & F_i^{\uparrow}(0)\frac{\partial\mathcal{T}(0,z)}{\partial z} - F_i^{\downarrow}(\infty)\frac{\partial\mathcal{T}(z,\infty)}{\partial z} \\ & + \Delta\tilde{\nu}_i \frac{\partial}{\partial z}\left[\int_0^z \pi\overline{B}_i(z')\frac{\partial\mathcal{T}_i(z',z)}{\partial z'}\,dz' \right. \\ & \left. + \int_z^\infty \pi\overline{B}_i(z')\frac{\partial\mathcal{T}_i(z,z')}{\partial z'}\,dz' \right] . \end{aligned} \qquad (10.58)$$

To evaluate the partial derivatives of the integral terms, in which z appears both as one of the limits of integration *and* as an argument to the integrand itself, we invoke the following mathematical identity:

$$\frac{\partial}{\partial x}\int_{x_0}^x f(x,y)\,dy \equiv \int_{x_0}^x \frac{\partial f(x,y)}{\partial x}\,dy + f(x,x) \,, \qquad (10.59)$$