Since the expressions on the left-hand sides of both (10.63) and (10.64) are equal to zero, we can subtract both expressions from inside the square brackets of (10.60) without changing the latter's validity. After considerable manipulation and cancellation of terms, we get

$$\mathcal{H}(z) = \frac{1}{\rho(z)C_p}\Bigg\{ \\
-\left[F_i^{\uparrow}(0) - \Delta\tilde{\nu}_i\pi\overline{B}_i(z)\right]\frac{\partial\mathcal{T}(0,z)}{\partial z} \quad (A) \\
+\left[F_i^{\downarrow}(\infty) - \Delta\tilde{\nu}_i\pi\overline{B}_i(z)\right]\frac{\partial\mathcal{T}(z,\infty)}{\partial z} \quad (B) \\
-\Delta\tilde{\nu}_i\pi\int_z^{\infty}\left[\overline{B}_i(z') - \overline{B}_i(z)\right]\frac{\partial^2\mathcal{T}_i(z,z')}{\partial z'\partial z}\,dz' \quad (C) \\
-\Delta\tilde{\nu}_i\pi\int_0^{z}\left[\overline{B}_i(z') - \overline{B}_i(z)\right]\frac{\partial^2\mathcal{T}_i(z',z)}{\partial z'\partial z}\,dz' \quad (D) \\
\Bigg\}. \tag{10.65}$$

Equation (10.65) certainly looks different than the mathematically equivalent (10.60), and it tells a different kind of story as well. Each of the partial derivatives represents the degree of *radiative coupling* between level z and some other part of the column. It characterizes the degree to which radiation *emitted* by one component is *absorbed* by the the atmosphere at z *and vice versa*.

Thus, for each of the lines (A)–(D), the difference inside the square brackets represents the imbalance between (a) radiation emitted from the remote location and reabsorbed at level z, and (b) energy lost by emission from level z and reabsorbed at the remote location. If the difference is positive, then the contribution to heating at level z is positive; negative implies cooling. I will emphasize the word **exchange** wherever it appears below in order to reinforce this idea of a *two-way* process.

Let's now interpret each line in turn:

Term (A) represents net heating/cooling through radiative **exchange** with the lower boundary. In the thermal IR band, we can usually take the surface to be black, in which case we can replace $F_i^{\uparrow}(0)$ with $\Delta\tilde{\nu}_i \pi \overline{B}_i(T_s)$, where T_s is the surface temperature. Since T_s is usually greater than $T(z)$, Term A usually represents a heating term.

Term (B) represents heating/cooling through radiative **exchange** with the top of the atmosphere (TOA). If we're working in the thermal IR band, then $F_i^{\downarrow}(\infty)$ is usually taken to be zero, in which case the exchange is strictly one-way. In the LW band, therefore, term (B) describes *cooling to space.* If, on the other hand, we're working with solar radiation, then $F_i^{\downarrow}(\infty)$ represents the incident flux of solar radiation at the TOA, and $\overline{B}(z) = 0$, so that term (B) describes heating through the direct absorption of solar radiation.

Terms (C) and (D) collectively represent radiative **exchanges** between level z and all other levels in the atmosphere z'. In order for the net effect of this exchange to be significant, there must be a large temperature difference between the two levels, *and* the radiative coupling (represented by the second derivatives of $\mathcal{T}$) must be strong. The coupling is strongest when the transmission $\mathcal{T}$ changes rapidly in response to changes in both z and z'. In regions of the spectrum for which the atmosphere is strongly absorbing (e.g., the middle of the CO_2 15 μm band), the coupling is strongest with levels z' that are very close to z and where the temperature difference is small; therefore the contribution of these wavelengths to local heating at z is negligible.

Note further that (C) represents **exchanges** between z and higher levels of the atmosphere; (D) represents **exchanges** with lower altitudes. In the middle of the troposphere, the temperature generally decreases in a quasi-linear fashion with increasing z. This implies that the cooling contribution by (C) will usually be offset to a large degree by heating from (D). The reverse applies in the stratosphere, where temperature tends to increase with height. It follows, therefore, that heating/cooling due to exhanges with other levels will be strongest where there is a minimum or maximum in $T(z)$. In particular, the tropopause level "sees" radiation arriving from the warmer stratosphere (term C) as well as from warmer levels of the troposphere (term D); hence, the tropopause experiences positive heating contributions from both (C) and (D).

In the longwave band, every term except (B) represents radiative exchanges between levels having temperatures falling somewhere in the range 190–310 K. The typical temperature *difference* between two levels that are strongly coupled is on the order of 10s of K or less. Furthermore, as noted above, terms (C) and (D) tend to partly cancel each other, except in the vicinity of the tropopause.

Term (B), by contrast, represents a direct loss of radiation to space, with no compensating return radiation. This term is therefore always negative, and it is quite often the largest term overall in the longwave radiation budget, especially at altitudes for which the atmosphere rapidly becomes more transparent with increasing height (e.g., due to the rapid narrowing of absorption lines). *To a very good approximation in many cases, radiative cooling profiles in the atmosphere can be estimated from term (B) alone.* This is called the *cooling-to-space approximation*.

Model Atmospheres

It's nice to have an equation like (10.65) to tell you *how* heating or cooling at a particular level is physically related to profiles of temperature and band transmittance. But simply staring at it won't tell you much about the actual magnitude of the heating or cooling to expect at any particular level z. You have to write a program to numerically evaluate the various terms in the equation using suitable band transmittance models, and you then have to apply your program to a particular profile of atmospheric temperature, humidity, and trace gas composition.

Although you could, in principle, pull any old radiosonde sounding off the Internet and run your program on it (although some assumptions would still be required concerning ozone profiles and the like), atmospheric scientists usually like to start by running their radiative transfer codes on idealized profiles called *model atmospheres.* These don't represent actual observations but are designed to reflect typical atmospheric conditions for a particular location and season. The use of standard model atmospheres has