CHAPTER 11

The Radiative Transfer Equation With Scattering

Throughout this book so far, we have discussed scattering primarily as one of two mechanisms for *extinguishing* radiation, the other mechanism being absorption. Thus, the extinction coefficient β_e could be decomposed into the sum of the absorption coefficient β_a and scattering coefficient β_s. The single scatter albedo $\tilde{\omega}$, defined as $\beta_\mathrm{s}/\beta_\mathrm{e}$, was introduced as a convenient parameter describing the relative importance of absorption and scattering: when $\tilde{\omega} = 0$, extinction of radiation is entirely by way of absorption; when $\tilde{\omega} = 1$, then there is no absorption, only scattering.

When radiation is extinguished via scattering, its energy is not converted to another form; rather, the radiation is merely redirected. The *loss* of radiation along one line-of-sight due to scattering is therefore always associated with a *gain* in radiation along other lines-of-sight passing through the same volume.

You can easily observe the above phenomenon with the help of a powerful, narrow beam of light, such as that from a searchlight, an automobile headlight or a laser pointer. When the air is very clear — i.e., free of smoke, dust, haze, or fog — then the beam can pass directly in front of you and you will not see it, because essentially none of the radiation is scattered out of the original path into the direction toward your eyes. But if the air contains suspended particles, then these particles will scatter some fraction of the beam into all directions, including toward your eyes, and the path of the beam will be clearly apparent, especially against a dark background. In this case, scattering clearly serves as a *source* of radiation, as seen from your vantage point. Of course the original beam is depleted by the same process. For example, if a fog is thick enough, the headlights of an oncoming car can't be seen at all until it is relatively close to you.

This chapter introduces the terminology and mathematical notation required to account for scattering as a *source* of radiation in the radiative transfer equation.

11.1 When Does Scattering Matter?

When scattering is important as a source of radiation along a particular line-of-sight, then the complexity of calculations of radiative transfer along that line-of-sight greatly increases compared with the nonscattering case. This is because one must, in the worst case, solve for the intensity field not just in *one direction* along a *one-dimensional path* but for *all directions* simultaneously in *three-dimensional space*! You would therefore like to be able to neglect scattering (as a source, at least) whenever you can get away with it.

In fact, you can safely ignore scattering as a source whenever gains in intensity due to scattering along a line-of-sight are negligible compared with (a) losses due to extinction *and* (b) gains due to thermal emission. In the atmosphere, these conditions are *usually* satisfied for radiation in the thermal IR band and for microwave radiation when no precipitation (e.g., rain, snow, etc.) is present. In addition, if one is concerned only with the depletion of *direct* radiation from an isolated, point-like source, such as the sun, then the above conditions are usually satisfied to reasonable accuracy.

For virtually any problem involving the interaction of shortwave (ultraviolet, visible, and near-IR) radiation with the atmosphere, scattering is the dominant atmospheric source of radiation along any line-of-sight other than that looking directly at the sun. The blue sky, white or gray clouds, the atmospheric haze that reduces the visual contrast of distant objects — all of these make their presence known primarily by way of scattered radiation.

11.2 Radiative Transfer Equation with Scattering

11.2.1 Differential Form

Previously, we derived Schwarzschild's Equation (8.4) under the assumption that scattering was unimportant and that therefore $\beta_e = \beta_a$. Under that assumption, we found that the change in intensity dI along an infinitesimal path ds could be written as

$$dI = dI_{\text{abs}} + dI_{\text{emit}} \,, \tag{11.1}$$

where the depletion due to absorption is given by

$$dI_{\text{abs}} = -\beta_a I \; ds \,, \tag{11.2}$$

and the source due to emission is

$$dI_{\text{emit}} = \beta_a B(T) \; ds \,. \tag{11.3}$$

In order to generalize the equation to include scattering, we must recognize that depletion occurs due to both absorption and scattering, so that β_e rather than β_a must appear in the depletion term. Moroever, we must now add a source term that describes the contribution of radiation scattered *into* the beam from other directions, so that

$$dI = dI_{\text{ext}} + dI_{\text{emit}} + dI_{\text{scat}} \,, \tag{11.4}$$

where

$$dI_{\text{ext}} = -\beta_e I \; ds \,. \tag{11.5}$$

The term dI_{scat} requires more thought. First, we know it must be proportional to the scattering coefficient β_s, since without scattering there can be no contribution from this term. Second, we recognize that radiation passing through our infinitesimal volume from *any* direction $\hat{\mathbf{\Omega}}'$ can potentially contribute scattered radiation in the direction of interest $\hat{\mathbf{\Omega}}$. Moreover, these contributions from all directions will sum in a linear fashion — that is, the path taken by a photon arriving from one direction is not influenced by the presence of, or paths taken by, other photons.

Mathematically, these ideas are expressed as follows:

$$dI_{\text{scat}} = \frac{\beta_s}{4\pi} \int_{4\pi} p(\hat{\mathbf{\Omega}}', \hat{\mathbf{\Omega}}) I(\hat{\mathbf{\Omega}}') \; d\omega' \; ds \,, \tag{11.6}$$