11.2.3 Plane Parallel Atmosphere

Although we know that the atmosphere is far from horizontally homogeneous, especially where clouds are concerned, most analytic solutions and approximations to the radiative transfer equation with scattering have been derived for the plane parallel case. Why? There are three basic reasons:

- Plane-parallel geometry is really the only semi-realistic case that lends itself to straightforward analysis and / or numerical solution (e.g., in climate and weather forecast models).

- There are indeed problems (e.g., the cloud-free atmosphere, horizontally extensive and homogeneous stratiform cloud sheets) for which the plane-parallel assumption usually seems quite reasonable as an approximation to reality.

- Even where it is not reasonable, there remains considerable doubt about the best way(s) to handle three-dimensional inhomogeneity, especially when computational efficiency is essential. Therefore investigators tend to fall back on plane-parallel geometry (with minor embellishments, such as the so-called *independent pixel approximation*), knowing that it is not perfect but believing it to be better than nothing at all (this is fine, as long as the potential for large errors is understood by all concerned!).

To adapt (11.10) to a plane-parallel atmosphere, we reintroduce the optical depth τ, measured from the top of the atmosphere, as our vertical coordinate, and we will henceforth use $\mu \equiv \cos\theta$ to specify the direction of propagation of the radiation measured from zenith.[2] We then have

$$\mu \frac{dI(\mu, \phi)}{d\tau} = I(\mu, \phi) - J(\mu, \phi) , \tag{11.13}$$

[2]Some textbooks, such as L02 and S94, specify that $\mu \equiv \cos\theta$. Others, such as TS02, instead define $\mu \equiv |\cos\theta|$, as I also did in an earlier chapter of this book. When writing the equations of radiative transfer with scattering, each convention has its own advantages and disadvantages. Here I have chosen the definition that permits the same equation to be used for both upward and downward radiation.

where the source function for both emission and scattering is

$$J(\mu,\phi) = (1-\tilde{\omega})B + \frac{\tilde{\omega}}{4\pi}\int_0^{2\pi}\int_{-1}^{1} p(\mu,\phi;\mu',\phi')I(\mu',\phi')\, d\mu' d\phi' \; . \tag{11.14}$$

There is only a relatively small class of applications in which it is necessary to consider both scattering and emission at the same time. Two examples include (1) microwave remote sensing of precipitation, and (2) remote sensing of clouds near 4 μm wavelength, for which scattered solar radiation may be of comparable importance to thermal emission. Except where noted, the rest of this book will focus on problems involving scattering of solar radiation only, without the additional minor complication of thermal emission.

11.3 The Scattering Phase Function

One way to give physical meaning to the scattering phase function is to regard $\frac{1}{4\pi}p(\hat{\mathbf{\Omega}}', \hat{\mathbf{\Omega}})$ as a probability density: Given that a photon arrives from direction $\hat{\mathbf{\Omega}}'$ and is scattered, what is the probability that its new direction falls within an infinitesimal element $d\omega$ of solid angle centered on direction $\hat{\mathbf{\Omega}}$? The normalization condition (11.7) simply ensures that energy is conserved when there is no absorption ($\tilde{\omega} = 1$); i.e., the new direction of a scattered photon is guaranteed to fall *somewhere* within the available 4π steradians of solid angle, and you can't get more (or fewer) photons out than you put in.

The functional dependence of the phase function on $\hat{\mathbf{\Omega}}$ and $\hat{\mathbf{\Omega}}'$ can be quite complicated, depending on the sizes and shapes of the particles responsible for the scattering. Nevertheless, an important simplification can be made when particles suspended in the atmosphere are either spherical or else randomly oriented. For example, cloud droplets are spherical, and small aerosol particles and air molecules, while generally not spherical, have no preferred orientation.[3] In such cases, the scattering phase function for a volume of air depends only on the angle Θ between the original direction $\hat{\mathbf{\Omega}}$

[3]Falling ice crystals, snowflakes, and raindrops generally *do* have a preferred orientation due to aerodynamic forces, and this directional anisotropy must sometimes be considered in radiative transfer calculations.

and the scattered direction $\hat{\mathbf{\Omega}}'$, where

$$\boxed{\cos\Theta \equiv \hat{\mathbf{\Omega}}' \cdot \hat{\mathbf{\Omega}} \, .} \tag{11.15}$$

The ability to replace $p(\hat{\mathbf{\Omega}}', \hat{\mathbf{\Omega}})$ with $p(\hat{\mathbf{\Omega}}' \cdot \hat{\mathbf{\Omega}}) \equiv p(\cos\Theta)$ is very helpful, inasmuch as the number of independent directional variables needed to fully characterize p is reduced from four (two each for $\hat{\mathbf{\Omega}}$ and $\hat{\mathbf{\Omega}}'$) to only one. The normalization condition (11.7) then reduces to

$$\frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} p(\cos\Theta) \; \sin\Theta d\Theta d\phi = 1 \, , \tag{11.16}$$

or

$$\boxed{\frac{1}{2} \int_{-1}^{1} p(\cos\Theta) \; d\cos\Theta = 1 \, .} \tag{11.17}$$

Except where noted, this simplified notation for the phase function will be utilized throughout the remainder of this book.[4]

11.3.1 Isotropic Scattering

The simplest possible scattering phase function is one that is constant; i.e.

$$p(\cos\Theta) = 1 \, . \tag{11.18}$$

Scattering under this condition is known as *isotropic*. It describes the case that all directions $\hat{\mathbf{\Omega}}$ are equally likely for a photon that has just been scattered. Thus, the new direction the photon takes is in no way predictable from the direction it was traveling prior to being scattered; in other words, the photon "forgets" everything about its past.

An example of the random path of a single photon experiencing isotropic scattering is shown in Fig. 11.1a. Note that once the photon passes into the interior of the cloud layer it wanders aimlessly,

[4]It sometimes necessary, however, to recast a phase function that is inherently of the form $p(\cos\Theta)$ in terms of the absolute directions $(\hat{\mathbf{\Omega}}', \hat{\mathbf{\Omega}})$ in order to facilitate integration over zenith and/or azimuth angles θ and ϕ.

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