an exact reversal of direction with every scattering event, a special case that is imaginable but physically unlikely.

For isotropic scattering, as discussed in the previous subsection, we expect $g = 0$, since scattering into the forward and backward hemispheres is equally likely. This can be shown explicitly by substituting $p = 1$ into (11.20), expanding $d\omega$ in spherical polar coordinates as $\sin\theta d\theta d\phi$, and choosing $\hat{\mathbf{\Omega}} = \hat{z}$ so that the scattering angle Θ is the same as the zenith angle θ. Thus,

$$\begin{aligned} g &= \frac{1}{4\pi}\int_0^{2\pi}\int_{-\pi/2}^{\pi/2} \cos\theta \sin\theta \; d\theta \; d\phi \\ &= \frac{1}{2}\int_{-\pi/2}^{\pi/2} \cos\theta \sin\theta \; d\theta \\ &= \frac{1}{2}\int_{-1}^{1} \mu \; d\mu \\ &= 0 \; . \end{aligned} \tag{11.22}$$

Note that while $g = 0$ for isotropic scattering, other phase functions can also have $g = 0$ and not be isotropic. The best example is the Rayleigh phase function derived in section 12.2, which describes the scattering of radiation by particles much smaller than the wavelength.

For many problems of interest, such as scattering of solar radiation in clouds, the asymmetry parameter g falls in the range 0.8–0.9. In other words, cloud droplets are strongly *forward scattering* at solar wavelengths. Fig. 11.1b shows examples of photon paths for $g = 0.85$. Although the average distance traveled by a photon between scattering events is the same as for isotropic scattering (Fig. 11.1a), the photon is now far more likely to be scattered into a direction that is not too different from its previous direction of travel. As a result, the photon's path, while still random, is far less chaotic than the isotropic case. Statistically, the photon travels a much greater distance before experiencing a sharp reversal in course. It is therefore also more likely to reach the cloud base and less likely to exit at cloud top. In other words, we expect the diffuse transmittance to increase and the cloud-top albedo to decrease when the asymmetry is large.

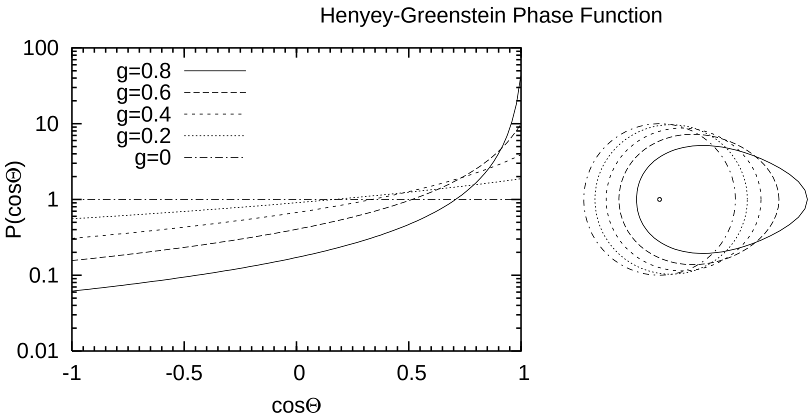

Fig. 11.2: The Henyey-Greenstein phase function plotted versus cos(Θ) (left) and as a log-scaled polar plot (right).

11.3.3 The Henyey-Greenstein Phase Function

The scattering phase functions of particles are often rather complicated (we will return to this subject in Chapter 12). As already pointed out, it is not always necessary to use a complete and accurate description of $p(\cos\Theta)$ in a radiative transfer calculation, as long as we know the asymmetry parameter g. For some types of calculations, we might want to employ a "stand-in" phase function that satisfies the following criteria:

- It should have a convenient mathematical form, ideally one that is an explicit function of the desired asymmetry parameter g.
- It should bear at least some resemblance to the shape of real phase functions, even if it doesn't have details like the rainbow, corona, etc. (See Chapter 12.)
- In order to be physically meaningful, the value of the phase function should be nonnegative for all values of Θ.

The *Henyey-Greenstein* phase function is the most widely used "model" phase function that satisfies all of the above criteria. It is given by

$$p_{\mathrm{HG}}(\cos\Theta) = \frac{1-g^2}{(1+g^2-2g\cos\Theta)^{3/2}} \,. \qquad (11.23)$$

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As you can see from Fig. 11.2, the HG phase function is isotropic for $g = 0$. For positive g, the function peaks increasingly in the forward direction but remains quite smooth. In other words, it captures the asymmetry of real phase functions rather well but not the higher order details.

Problem 11.1: Show that the parameter g appearing in (11.23) equals the asymmetry parameter as defined by (11.20).

Although the HG phase function with $g > 0$ does a good job of reproducing the observed forward peak in the phase functions of real particles, there is often also a pronounced (but somewhat smaller) backward peak which is *not* captured. Therefore, you will sometimes see the use of a *double* HG function, with one of the two terms serving to represent the backward peak:

$$p_{\mathrm{HG2}}(\cos\Theta) = b p_{\mathrm{HG}}(\cos\Theta; g_1) + (1-b) p_{\mathrm{HG}}(\cos\Theta; g_2) \ , \quad (11.24)$$

where $g_1 > 0$, $g_2 < 0$, and $0 < b < 1$.

Problem 11.2: (a) Given g_1, g_2, and b, find the asymmetry parameter g of the double Henyey-Greenstein phase function.

(b) For marine haze particles in the visible band, it has been found that good values for the above parameters are $b = 0.9724$, $g_1 = 0.824$, and $g_2 = -0.55$. Find g.

(c) Plot the phase function $p(\cos\Theta)$ described in part (b), using a logarithmic vertical axis.

11.4 Single vs. Multiple Scattering

When a solar photon enters the atmosphere or a cloud layer from the top, it will eventually either exit again (top or bottom) or else get absorbed. There are no other possibilities. Before either one happens, however, the photon may experience anywhere from zero to a very large number of scatterings from atmospheric particles.

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