I of an object and the brightness I' of its surroundings:

$$C \equiv \frac{I' - I}{I'} \,. \tag{11.37}$$

In a purely absorbing atmosphere, a mere reduction in transmittance along a line-of-sight has no impact on the visual contrast between two objects at the same distance, and therefore relatively little on visibility (up to the limit imposed by your eyes' sensitivity to light), as long as the fractional reduction in brightness is the same for both.

Atmospheric scattering reduces contrast by adding a source of radiation to the line-of-sight that is independent of the brightness of whatever is at the far end of the path. Since this source is integrated along the line-of-sight, a long path produces a greater reduction in contrast than a short path. The distance at which the contrast of an object is reduced to the minimum level required for visual detection defines the visibility.

Let's analyze the visibility problem quantitatively, by considering the contribution of single-scattered radiation to the radiance along a finite horizontal path s. Because of the latter condition, we can't use the plane-parallel form of the RTE but must start with an adaptation of (11.9):

$$\frac{dI}{d(\beta_e s)} = -I + J \,, \tag{11.38}$$

where I is the intensity measured horizontally in azimuthal direction ϕ, J is the scattering source function given by

$$J = \frac{\tilde{\omega}}{4\pi} \int_{4\pi} p(\mu_0, \phi_0; 0, \phi) I(\hat{\Omega}') \, d\omega' \,, \tag{11.39}$$

and s is the distance in the direction *toward* the observer.

For this problem, we can assume a horizontally homogeneous atmosphere, so that both the extinction coefficient β_e and the scattering source function J are constant along the line-of-sight. With these assumptions, we can integrate (11.38) to get

$$I(S) = I(0) e^{-\beta_e S} + \left(1 - e^{-\beta_e S}\right) J \,, \tag{11.40}$$

where $I(0)$ is the "intrinsic" radiance of the remote scene as seen without any intervening atmosphere, and $I(S)$ is the brightness of

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the same scene at the observer's distance S. We see that the observed intensity is just a weighted average of the intrinsic brightness of the distant object and the scattering source function, with the weight being the path transmittance $t = e^{-\beta_e S}$ for the first term and $1 - t$ for the second. Obviously, if $t = 0$, we see only the atmospheric scattering and no trace of the object at $s = 0$.

Problem 11.5: Fill in the steps of the derivation of (11.40) from (11.38). Hint: Multiplying both sides by an integrating factor $e^{\beta_e S}$ will allow you to recast the differential equation into a form that can be directly integrated.

Now let's use the above equation to compute the contrast of a black object with $I(0) = 0$ viewed against a white background with intensity $I'(0)$:

$$C = \frac{I'(S) - I(S)}{I'(S)} = \frac{I'(0)t}{I'(0)t + (1-t)J} \, . \qquad (11.41)$$

We are interested in the distance S corresponding to the minimum contrast that still permits the human eye to distinguish the object from its background, so we invert the above equation to get

$$S = \frac{1}{\beta_e} \ln \left[\frac{I'(0)(1-C)}{CJ} + 1 \right] \, . \qquad (11.42)$$

Now all that is left is to make reasonable assumptions about $I'(0)$, C, and J.

For the background, we assume an intensity $I'(0) = \alpha F_0$, where α depends on the reflective properties of the background for the particular viewing geometry and direction of the incident sunlight. For example, if the background is a nonabsorbing Lambertian reflector, then $\alpha \leq 1/\pi$, with the equality applying in the case of normal solar incidence.

As before, we'll assume that the atmosphere is optically thin in the vertical and that the sun is high in the sky, so the scattering source function can be approximated as

$$J \approx \frac{F_0 \tilde{\omega}}{4\pi} p(\mu_0, \phi_0; 0, \phi) \, , \qquad (11.43)$$

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where μ_0 is the cosine of the solar zenith angle. We'll assume that the phase function can be expressed in terms of the cosine of the scattering angle alone, with

$$\begin{aligned} \cos\Theta &= \hat{\mathbf{\Omega}}_0 \cdot \hat{\mathbf{\Omega}} \\ &= \left(\sqrt{1-\mu_0^2}\cos\Delta\phi, \sqrt{1-\mu_0^2}\sin\Delta\phi\,, \mu_0\right)\cdot(1,0,0) \\ &= \sqrt{1-\mu_0^2}\cos\Delta\phi \end{aligned} \tag{11.44}$$

where $\Delta\phi = \phi - \phi_0$ is the angle between the viewing azimuth and the solar azimuth.

We can now substitute the above expressions for J and $I'(0)$, with $\alpha \approx \mu_0 / \pi$ and $C \approx 0.02$, to get

$$S \approx \frac{1}{\beta_e} \ln\left[\frac{200\mu_0}{\tilde{\omega}p(\cos\Theta)} + 1\right] . \tag{11.45}$$

Problem 11.6: Use (11.45) together with the phase function for marine haze given in Problem 11.2 to plot the visibility in km versus azimuth $\Delta\phi$ relative to the sun's direction for two cases: $\mu_0 = 1$ and $\mu_0 = 0.5$. For both cases assume $\beta_e = 1.0\ \mathrm{km}^{-1}$. Explain the differences between the two curves. Are your results consistent with your experience?

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