the number of particles (per unit volume) having that radius and then integrated over all possible radii.

A completely analogous relationship gives the scattering coefficient

$$\beta_s = \int_0^\infty n(r) Q_s(r) \pi r^2 \, dr \; . \tag{12.27}$$

From there, we immediately have the single-scatter albedo of the distribution as $\tilde{\omega} = \beta_s / \beta_e$.

The combined scattering phase function is the *scattering cross-section weighted average* of the individual phase functions:

$$p(\cos\Theta) = \frac{1}{\beta_s} \int_0^\infty n(r) Q_s(r) \pi r^2 p(\cos\Theta; r) \, dr \; , \tag{12.28}$$

which also implies a combined asymmetry parameter of

$$g = \frac{1}{\beta_s} \int_0^\infty n(r) Q_s(r) \pi r^2 g(r) \, dr \; . \tag{12.29}$$

12.5 Applications to Meteorology, Climatology, and Remote Sensing

12.5.1 The Scattering Properties of Clouds

The radiative properties of clouds, including their ability to reflect and absorb both solar and thermal radiation, depend on their optical depth τ^*, their single scatter albedo $\tilde{\omega}$ and the scattering phase function $p(\cos\Theta)$. These properties in turn depend on the size parameter x and on the complex index of refraction m for the cloud's constituent particles. Both x and m depend on wavelength λ, and x also depends on the droplet radius r. The index of refraction m depends on composition and material phase as well, but for most clouds, there are only two possibilities: liquid water or ice.

For reasonably large x, we already saw that $Q_e \approx 2$, so that the optical thickness τ^* can often be taken to be almost independent of wavelength. The phase function $p(\Theta)$ is adequately characterized for many purposes by the asymmetry parameter g, which we saw tends to hover in the fairly narrow range 0.8–0.9 for x greater than around 10.

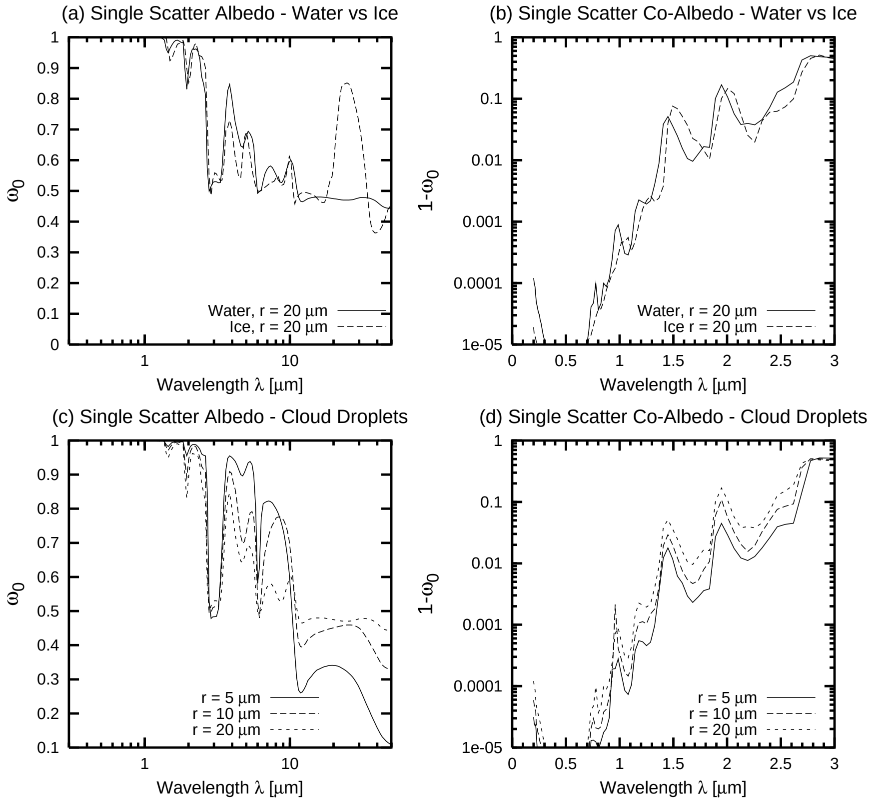

Fig. 12.10: Single scatter albedo (or co-albedo) as a function of wavelength for water and ice spheres of various sizes. The left column depicts $\tilde{\omega}$ over the entire visible, near IR and thermal IR range. The right column depicts the scattering co-albedo (defined as $1 - \tilde{\omega}$) for just the solar band. The top row compares water and ice for particle radius $r = 20\ \mu$m; the bottom row compares water droplets of three different radii (5, 10, and 20 μm).

This leaves the single scatter albedo $\tilde{\omega}$ as the one variable that could potentially have a large influence on how cloud reflectivity/absorptivity varies with λ. And indeed this conjecture is validated by the plots of $\tilde{\omega}$ vs. λ shown in Fig. 12.10. This information is represented in two different ways. The first is by simply plotting $\tilde{\omega}$ on a linear vertical axis, as is done in the two panels in the left column. This is fine for showing the coarse variations of $\tilde{\omega}$ with wavelength but tends to obscure subtle deviations of $\tilde{\omega}$ from exactly one (pure scattering, no absorption), which can nevertheless be significant for absorption by clouds. Therefore, for the shorter wavelengths where absorption is comparatively weak, we plot the scattering *co-albedo*, defined as $1 - \tilde{\omega}$, on a logarithmic vertical axis.

Here are the basic points you should take away from these plots:

- The visible band ($0.4\ \mu\text{m} < \lambda < 0.7\ \mu\text{m}$) coincides almost exactly with a surprisingly narrow portion of the EM spectrum for which absorption by cloud droplets is, for all practical purposes, zero. You can think of it as an astonishing coincidence that clouds (when viewed from the sunlit side) appear to our eyes as white rather than gray, black, or some other color! As soon as you move into either the UV or near-IR bands, $\tilde{\omega}$ quickly decreases to well below 1, settling into the range 0.5–0.8 for most of the IR band. For even $\tilde{\omega} = 0.8$, the albedo of a thick cloud is only around 15%.

- At several wavelengths, there is a significant difference between the single scatter albedo of a spherical ice particle and that of a water droplet of the same size (top row). For some of these wavelengths, ice particles are less absorptive than the water droplets; for others, the reverse is true. These differences can be exploited by satellite sensors to distinguish ice phase clouds (cirrus) from water clouds[9].

- For most wavelengths, there is a significant dependence of the single scatter albedo on the droplet radius in liquid water clouds (bottom row). As a general rule (although there are exceptions), a larger droplet has lower $\tilde{\omega}$ (i.e., is more absorptive) than a smaller droplet at the same wavelength. Once again, satellite remote sensing techniques can exploit this property to estimate the effective droplet radius r_{eff} in water clouds.

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[9] The fact that ice particles in clouds are generally *not* spheres complicates the problem somewhat, but the principle is still valid.