(12.10). Substituting these into (12.33) gives

$$Q_b = 4x^4 \left| \frac{m^2 - 1}{m^2 + 2} \right|^2 . \quad (12.36)$$

If our particles are too large, then Rayleigh theory no longer applies, and we have to calculate σ_b using Mie theory. Fig. 12.11 shows accurate calculations of Q_b for a wide range of sizes of water and ice spheres. The wavelength $\lambda = 10.71$ cm chosen for these calculations corresponds to that used by the current-generation operational weather radar network in the United States.

You can see that for liquid water spheres up to a diameter of about 6 mm (solid curve), the Rayleigh relationship (12.36) holds to a high degree of accuracy: each decade (factor ten) increase in D corresponds to a four decade (factor 10^4) increase in Q_b. In fact, 6 mm corresponds to a rough upper limit on the observed sizes of raindrops in heavy rain; beyond this size, raindrops tend to be broken up by aerodynamic forces as they fall.

Hailstones can of course become considerably larger than raindrops. It is therefore convenient that the Rayleigh approximation apparently holds up to a diameter of around 3 cm for a pure ice sphere (dashed curve). Note that for any given D in the Rayleigh regime part of the curve, Q_b for pure ice is only 20% of that for liquid water. This difference is due to the substantially smaller value of m for ice in the microwave band, as compared to liquid water[10].

Problem 12.5: Based on the above information, compute the diameter of a spherical hailstone that has the same radar backscatter cross-section σ_b (not Q_b!) as a spherical raindrop with a diameter of 2 mm.

Let's assume that the hydrometeors (e.g. raindrops, hailstones, etc.) that are observed by a 10-cm weather radar all fall in the Rayleigh regime. We can then substitute (12.36) into (12.35) to get

$$\eta = \frac{\pi^5}{\lambda^4} \left| \frac{m^2 - 1}{m^2 + 2} \right|^2 \int_0^\infty n(D) D^6 \, dD . \quad (12.37)$$

[10] It is worth keeping in mind, however, that growing hailstones often have a coating of liquid water. Even a thin coating of water can drastically alter the radar backscattering properties of an ice particle.

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Substituting this expression back into (12.31), we find that the backscattered power measured by the radar receiver is

$$P_r \propto \left| \frac{m^2 - 1}{m^2 + 2} \right|^2 \frac{Z}{d^2} , \tag{12.38}$$

where Z is the *reflectivity factor*, defined as

$$\boxed{Z = \int_0^\infty n(D) D^6 \, dD .} \tag{12.39}$$

In other words, the reflectivity factor is numerically equal to *the sum of the sixth powers of the diameters of all of the drops in a unit volume of air.* The standard units of Z used by meteorologists are [$\mathrm{mm}^6\mathrm{m}^{-3}$]. *An estimate of the reflectivity factor Z at each range d along the beam is what most weather radars record and display.*

Because observed values of Z span an enormous range, meteorologists prefer to work with a logarithmic representation of Z, defining a nondimensional unit dBZ, which means "decibels with respect to one standard unit of Z." You convert the reflectivity factor from standard units to units of dBZ as follows:

$$Z\,[\mathrm{dB}Z] = 10 \log_{10}(Z) , \tag{12.40}$$

where Z on the right hand side is the numerical value of Z expressed in standard (dimensional) units of reflectivity. Thus, an increase in reflectivity by 10 dBZ corresponds to a factor ten increase in Z expressed in standard units. An increase of 30 dBZ implies a thousand-fold increase in reflectivity.

Problem 12.6: Depending on range, a typical weather radar can measure reflectivities from as low as -20 dBZ to as high as 70 dBZ. In terms of physical units, what is the ratio of the two reflectivity factors?

In converting the received power P_r to an estimate of the reflectivity factor Z, the radar processing software assumes a value of

m appropriate to liquid water in (12.38). The displayed quantity is therefore actually better regarded as an *equivalent reflectivity factor* Z_e which may or may not be equal to the *true* reflectivity factor Z defined by (12.39), depending on whether the targets are liquid water or something else, like ice. If the particles are in fact ice, then

$$Z_e \approx 0.20Z \ . \tag{12.41}$$

Problem 12.7: During a particular (and peculiar) rainstorm, each cubic meter of air contains 1000 falling drops, each of identical diameter D. (a) Compute the reflectivity factor Z, assuming $D = 1$ mm. (b) Repeat for $D = 2$ mm. (c) By what factor did Z increase on account of a mere two-fold increase in D? (d) Express your answers to (a)–(c) in units of dBZ. (e) If you replace the liquid raindrops with ice spheres of the same size, by how many dBZ will the radar-estimated effective reflectivity Z_e be reduced? (f) Notwithstanding Eq. (12.41), hailstorms are often recognized on radar displays by virtue of their anomalously *high* Z_e. Why?

In actual rainfall, drops do not all have one size but rather are distributed over a wide range of sizes. Because of the D^6 dependence in Z, observed reflectivities are heavily influenced by the few largest drops in the volume of air. A single drop with a diameter 5 mm reflects more microwave radiation than 15,000 drops of 1 mm diameter! And clouds, with their typical droplet diameters of around 20 μm, are completely invisible to all but the most sensitive radars, despite typical droplet concentrations in excess of 10^8 m^{-3}.

Problem 12.8: From the information given above concerning cloud droplets, find a typical reflectivity factor Z for clouds, expressed in dBZ.

Radar Rainfall Estimation

Raindrops passing through the air eventually reach the surface, and the rate at which water is deposited (depth per unit time) is known