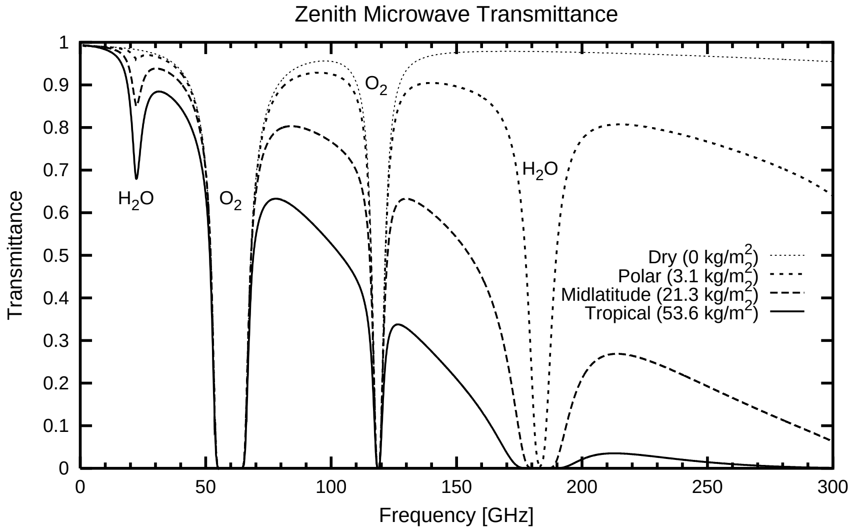

Fig. 12.13: Zenith microwave transmittance of the cloud-free atmosphere for different models of atmospheric temperature and humidity. The vertically integrated water vapor content associated with each model is given in parentheses.

small that we can ignore scattering, and the mass extinction (absorption) coefficient k_L of cloud liquid water is accurately given by (12.19). Figure 12.12 shows how k_a varies with frequency over the microwave band.

Consider a microwave radiometer at ground level viewing vertically incident radiation emitted by the atmosphere. In the microwave band, the Rayleigh-Jeans approximation allows us to work with brightness temperature T_B as a convenient stand-in for radiant intensity, with $T_B = \varepsilon T$, where ε is the emissivity of a surface or atmospheric layer, and T is its physical temperature (see section 6.1.4).

If we assume for the moment that the cloud-free atmosphere is perfectly transparent (it is not) and that there is a single cloud layer with average temperature T and total vertically integrated cloud liquid water L, then the measured brightness temperature is given approximately by

$$T_B = \varepsilon T = [1 - t(L)]\, T = [1 - \exp(-k_L L)]\, T \; . \qquad (12.44)$$

You could then use your upward-looking microwave radiometer to

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estimate the cloud water path by simply (i) solving the above equation for L, (ii) assuming something reasonable for T, and (iii) plugging in the observed brightness temperature T_B.

The reality is of course slightly more complicated. In particular, there are two other atmospheric constituents that always contribute additional absorption and emission in the microwave band: water vapor, and oxygen (Fig. 12.13). If we stay well away from the 60 GHz and 118 GHz absorption bands due to oxygen, then the reduction in transmittance due to the dry atmosphere alone is only a few percent. Furthermore, since the surface air pressure at any given location, and therefore the total column oxygen content, varies only by about 5%, we can get away with assuming a fixed optical depth τ_O due to oxygen.

Water vapor is a bigger problem, because atmospheric column vapor content V varies from very low (~ 1 kg m^{-2}) in dry polar air masses to rather high (up to 60 kg m^{-2}) in humid tropical air masses. In order to limit the total optical depth due to water vapor, let's confine our attention to the spectrum below about 40 GHz, so that even in the worst case, we still have a zenith transmittance of at least 60% or so. That way, the atmosphere will never become so opaque due to water vapor that it becomes hard to see changes in opacity due to cloud water.

If we assume that the mean emitting temperature of the atmospheric water vapor and oxygen isn't *too* different from that of the cloud layer, then we can write

$$T_B \approx [1 - \exp(-\tau)]\, T \,, \tag{12.45}$$

where the total atmospheric optical depth is approximated as

$$\tau \approx \tau_O + k_L L + k_V V \,, \tag{12.46}$$

and k_V is the column-averaged mass absorption coefficient of water vapor. We can divide through by T, rearrange, and take the logarithm of both sides to get

$$y \equiv \log\left(\frac{T - T_B}{T}\right) \approx -k_L L - k_V V - \tau_O \,. \tag{12.47}$$

Given a reasonable value for T, the new variable y is a known function of the observed T_B. The definition of y is convenient because it turns out to be a simple linear function of our two unknowns V and L. Unfortunately, we have one equation in two unknowns, and so a measurement of T_B at a single wavelength is not sufficient to uniquely determine both variables.

Let's therefore design our radiometer to measure T_B at two different frequencies ν_1 and ν_2. We can then write our equation in matrix form as

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = -\begin{bmatrix} k_{L,1} & k_{V,1} \\ k_{L,2} & k_{V,2} \end{bmatrix} \begin{bmatrix} L \\ V \end{bmatrix} - \begin{bmatrix} \tau_{O,1} \\ \tau_{O,2} \end{bmatrix} . \tag{12.48}$$

We now have two linear equations in two unknowns. In principle, we can solve for L and V as follows:

$$\begin{bmatrix} L \\ V \end{bmatrix} = -\begin{bmatrix} k_{L,1} & k_{V,1} \\ k_{L,2} & k_{V,2} \end{bmatrix}^{-1} \begin{bmatrix} y_1 + \tau_{O,1} \\ y_2 + \tau_{O,2} \end{bmatrix} , \tag{12.49}$$

assuming that the inverse of the matrix of absorption coefficients $\mathbf{K}$ exists.

Mathematically speaking, the inverse exists if the determinant $\|\mathbf{K}\| \neq 0$, a condition that is almost guaranteed to be satisfied for any pair of distinct microwave frequencies. *Practically* speaking, however, the mere existence of an inverse is not enough! Why not?

Recall that (12.47) was presented as an *approximate* model of the dependence of y on L and V. This suggests that we should modify (12.49) to allow for the likelihood of errors ϵ_i in the relationship:

$$\begin{bmatrix} L' \\ V' \end{bmatrix} = -\begin{bmatrix} k_{L,1} & k_{V,1} \\ k_{L,2} & k_{V,2} \end{bmatrix}^{-1} \left\{ \begin{bmatrix} y_1 + \tau_{O,1} \\ y_2 + \tau_{O,2} \end{bmatrix} - \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \end{bmatrix} \right\} , \tag{12.50}$$

where L' and V' are now *estimates* of the true L and V. The *estimation error* can then be written

$$\begin{bmatrix} L' - L \\ V' - V \end{bmatrix} = \begin{bmatrix} k_{L,1} & k_{V,1} \\ k_{L,2} & k_{V,2} \end{bmatrix}^{-1} \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \end{bmatrix} . \tag{12.51}$$

The goal of a remote sensing technique, of course, is to make sure that the estimation errors are as small as possible. In this instance, it means ensuring that the matrix $\mathbf{K}^{-1}$ does not excessively "amplify" the model and / or instrument errors ϵ_i. Since the magnitude of $\mathbf{K}^{-1}$ is proportional to $1/\|\mathbf{K}\|$, it follows that we require not only that