CHAPTER 13

Radiative Transfer with Multiple Scattering

We are now nearing the end of our introductory survey of radiative transfer in the atmosphere, and it is perhaps no surprise that we have saved the best — or at least the most challenging — for last. Every problem we have dealt with so far entailed either absorption and emission with no scattering, or else at most single scattering. These restrictions enabled us to solve the radiative transfer equation along a single line of sight without worrying about what was going on in other locations and directions.

Such simplifications are utterly useless for solar radiative transfer in clouds. Because most water clouds are both optically thick ($\tau \gg 1$) and are only weakly absorbing ($\tilde{\omega} \approx 1$), multiple scattering cannot be neglected. That is to say, at most points in the interior of the cloud, the majority of radiation incident on a cloud particle will have already been scattered at least once by some other particle. Photons incident at cloud top will typically be scattered numerous times before re-emerging from the cloud, either at the top or base in the plane parallel case, or even from the sides in the case of three-dimensional clouds. What this means in practice is that you cannot consider what is happening to the radiant intensity along one line-of-sight without simultaneously considering what it is doing everywhere else.

The full radiative transfer equation for a plane parallel atmosphere was given earlier as (11.13) and (11.14). If we neglect thermal emission, the two can be combined to give

$$\mu \frac{dI(\mu,\phi)}{d\tau} = I(\mu,\phi) - \frac{\tilde{\omega}}{4\pi}\int_0^{2\pi}\int_{-1}^{1} p(\mu,\phi;\mu',\phi')I(\mu',\phi')\, d\mu' d\phi' \, . \tag{13.1}$$

This integrodifferential equation tells us that in order to determine $I(\mu,\phi)$ at a particular level τ in a cloud, we must simultaneously determine $I(\mu',\phi')$ for all values of μ' and ϕ' and for all other values of τ.

In general, (13.1) cannot be solved exactly except under extremely restrictive (and inevitably unrealistic) assumptions about the scattering phase function, among other things. Therefore, radiation specialists have put much effort into

- studying closed form solutions to (13.1) for highly idealized cases (e.g., isotropic scattering, infinite cloud optical depth) with an eye toward gaining *qualitative* insight into radiative transfer in clouds, and

- developing computational techniques for obtaining reasonably accurate *numerical* solutions for real-world problems.

We will not delve deeply into such methods here, as they are mainly of interest to those who perform radiative transfer calculations for a living. If this book has piqued your own interest in atmospheric radiation, then you should plan to continue your education with advanced textbooks, such as L02 and TS02, that devote considerable space, and literally hundreds of equations, to computational methods.

Here, we will begin by trying to convey some insight into how multiple scattering "works" in a plane-parallel cloud layer. We will then walk through one of the simplest possible analytic solutions to (13.1) known as the *two-stream method.* The two-stream approximation is not terribly useful for computing accurate radiant intensities as a function of μ and ϕ, but it's not bad for estimating hemispherically averaged *fluxes* in plane parallel cloud layers. In fact, because it is relatively undemanding of computer resources, some variation

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