properties of clouds, the interpretation of your results is easiest if there are no other variables to consider. If a cloud is optically thin, what you see from above the cloud layer is at least as sensitive to the radiative properties of the surface below the cloud as it is to the cloud properties themselves.

To exclude such influences, we will first consider the case of a *semi-infinite* cloud; i.e., a cloud layer with an upper boundary at $\tau = 0$ but which is effectively infinite in depth below that level. A cloud need not be semi-infinite in a literal sense in order to behave radiatively like a semi-infinite cloud; all that's necessary is for it be so thick that a photon incident on the cloud top has essentially zero chance of emerging from the bottom before either getting absorbed or else getting scattered back up through the cloud top.

We adapt (13.39) and (13.40) to the case of a semi-infinite cloud simply by letting $\tau^* \to \infty$, which gives us

$$I^{\uparrow}(\tau) = I_0 r_\infty e^{-\Gamma\tau} \, , \tag{13.41}$$

$$I^{\downarrow}(\tau) = I_0 e^{-\Gamma\tau} \, . \tag{13.42}$$

13.3.1 Albedo

Armed with (13.41) and (13.42), we can look at several interesting radiative properties of our cloud. Let's start by finding the albedo at cloud top, which is defined as the ratio of the reflected to incident radiation flux:

$$\text{Albedo} = \frac{\pi I^{\uparrow}(0)}{\pi I^{\downarrow}(0)} = \frac{I_0 r_\infty e^{-\Gamma\tau}}{I_0 e^{-\Gamma\tau}} \, , \tag{13.43}$$

which simplifies to

$$\boxed{\text{Albedo} = r_\infty \, .} \tag{13.44}$$

We discover that the albedo of a semi-infinite cloud is just r_∞, which explains (retroactively) why that particular function of $\tilde{\omega}$ and g was singled out for its own special symbol.

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With that fact in mind, let's study the properties of r_∞ more closely. For convenience, its definition is repeated here:

$$\boxed{r_\infty \equiv \frac{\sqrt{1-\tilde{\omega}g} - \sqrt{1-\tilde{\omega}}}{\sqrt{1-\tilde{\omega}g} + \sqrt{1-\tilde{\omega}}}\,.} \tag{13.45}$$

For starters, if $\tilde{\omega} = 1$, then $r_\infty = 1$, regardless of the value of g (as long as $g < 1$). This makes sense, because if there is zero absorption, then any photon incident on the top of a semi-infinite cloud must eventually emerge from the top again, no matter how many times it gets scattered first. It cannot get permanently lost deep inside the cloud, because no matter where it is, the photon is still closer to the top than it is to the (infinitely distant) bottom and therefore its random wanderings are statistically guaranteed to take it to the top eventually.

You can also see that if $g = 1$, then $r_\infty = 0$, regardless of the value of $\tilde{\omega}$ (as long as $\tilde{\omega} < 1$). Again, this makes sense, because in this case, every photon that is "scattered" continues traveling in exactly the same direction as before and can never change direction to return to the surface. However, this case is unrealistic for two reasons: (i) g is *always* less than one for any real scattering medium, and (ii) even if g *were* equal to one, then you might as well say that the medium doesn't scatter, since all "scattered" radiation continues traveling in its original direction as if it had never been scattered in the first place.

Having dealt with those two limiting cases, let's consider the more realistic situation in which $g < 1$ and $0 < \tilde{\omega} < 1$. Fig. 13.4 shows how r_∞ varies with $\tilde{\omega}$ for two different values of g, the larger value ($g = 0.85$) being typical of real clouds in the solar band. The albedo is zero for $\tilde{\omega} = 0$ and goes to one for $\tilde{\omega} \rightarrow 1$, as expected. The slighly less obvious point to note is that the overall absorptivity of the cloud, which is given in this case by one minus the albedo (since transmittance is zero for a semi-infinite cloud), is quite significant even for $\tilde{\omega}$ fairly close to one.

For example, $\tilde{\omega} = 0.999$ and $g = 0.85$ yields $r_\infty = 0.85$, corresponding to a cloud absorptivity of 15%. In other words, even though there is only a very slight chance for a photon to get absorbed in any *single* extinction event (in this example, that chance is

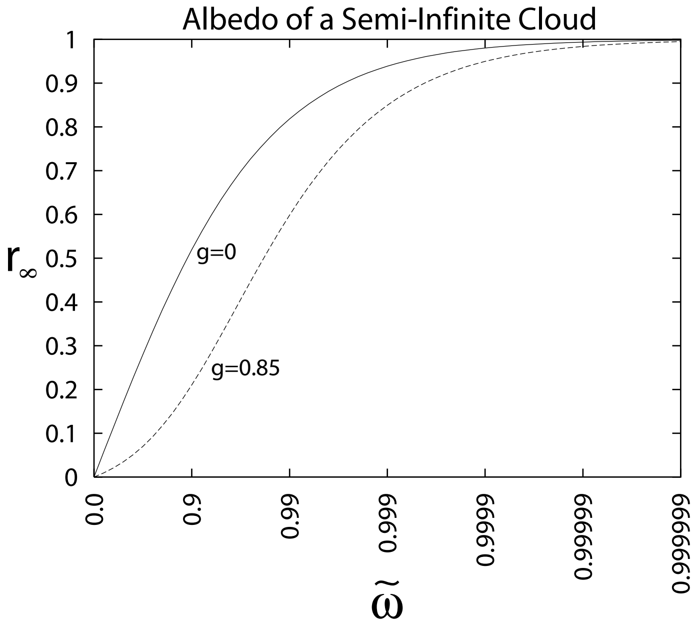

Fig. 13.4: The albedo of a semi-infinite cloud, as computed from (13.45).

only 0.1%), there is a much greater probability (15%) that a photon incident on the top of the cloud will get absorbed at some point in its wanderings before emerging from the cloud top again. The reason, of course, is that the photon's probability of survival is equal to $\tilde{\omega}^n$, where n is the number of scattering events it experiences inside the cloud. For a deep cloud with small scattering co-albedo (i.e., $1 - \tilde{\omega} \ll 1$), n can be a fairly large number.

Problem 13.2: Assume that $r_\infty = \tilde{\omega}^{\overline{n}}$, where $\overline{n}$ is the *effective mean number of scatterings* that photons incident on a semi-infinite cloud undergo inside the cloud before reemerging from the cloud top. (a) For the case that $\tilde{\omega} = 0.9999$ and $g = 0.85$, compute r_∞ and $\overline{n}$. (b) Repeat the above calculation, but with $\tilde{\omega} = 0.9$. (c) Explain why $\overline{n}$ is much different for the above two cases.