13.4 Nonabsorbing Cloud

Let's now abandon our semi-infinite cloud, and turn to the more realistic case of cloud layer with finite optical depth τ^*. We will initially consider the case that scattering is *conservative*; i.e., $\tilde{\omega} = 1$. This assumption sounds drastic, but in fact the single scatter albedo of cloud droplets is very close to one over most of the visible band (see Fig. 12.10), and absorption by clouds is indeed negligible for most purposes within that band.

Note that if we simply set $\tilde{\omega} = 1$ and then try to evaluate (13.39) and (13.40) we run into the problem that $\Gamma = 0$. Each equation then collapses to the ratio $0/0$, which is undefined. The obvious workaround is to take the limit of each equation as $\tilde{\omega} \to 0$. But there is an easier way: let's go back to an earlier step in our original derivation, namely (13.18) and (13.19). We can now re-solve these from scratch, using $\tilde{\omega} = 1$. Equation (13.18) then becomes

$$\frac{1}{2}\frac{d}{d\tau}(I^{\uparrow} - I^{\downarrow}) = 0\,, \tag{13.52}$$

which implies

$$I^{\uparrow} - I^{\downarrow} = \text{constant} \quad \rightarrow \quad \pi(I^{\uparrow} - I^{\downarrow}) = F^{\text{net}} = \text{constant}\,. \tag{13.53}$$

or

$$I^{\uparrow} - I^{\downarrow} = \frac{F^{\text{net}}}{\pi} = \text{constant}\,. \tag{13.54}$$

In other words, the net flux does not change with depth in the cloud. This is what you would expect, because a change of F^{net} would imply absorption (and heating), and there can be no absorption when $\tilde{\omega} = 1$. Similarly, (13.19) becomes

$$\frac{d}{d\tau}(I^{\uparrow} + I^{\downarrow}) = 2(1-g)(I^{\uparrow} - I^{\downarrow}) = \frac{2F^{\text{net}}}{\pi}(1-g)\,, \tag{13.55}$$

which integrates to

$$I^{\uparrow} + I^{\downarrow} = \frac{2F^{\text{net}}\tau}{\pi}(1-g) + K\,, \tag{13.56}$$

F^{net} and K are constants of integration whose values will be determined by the boundary conditions. Solving (13.54) and (13.56) for

$I^{\uparrow}$ and $I^{\downarrow}$ gives

$$I^{\uparrow} = \frac{F^{\text{net}}}{2\pi}[1 + 2\tau(1-g)] + \frac{K}{2} \,, \tag{13.57}$$

and

$$I^{\downarrow} = -\frac{F^{\text{net}}}{2\pi}[1 - 2\tau(1-g)] + \frac{K}{2} \,. \tag{13.58}$$

We now apply the same boundary conditions as before: $I^{\uparrow}(\tau^*) = 0$, and $I^{\downarrow}(0) = I_0$, giving us

$$\frac{K}{2} = I_0 + \frac{F^{\text{net}}}{2\pi} \,, \tag{13.59}$$

and

$$F^{\text{net}} = \frac{-\pi I_0}{1 + (1-g)\tau^*} \,. \tag{13.60}$$

The general solution of the two-stream equations for the case of conservative scattering is then

$$\boxed{I^{\uparrow}(\tau) = \frac{I_0(1-g)(\tau^* - \tau)}{1 + (1-g)\tau^*} \,,} \tag{13.61}$$

$$\boxed{I^{\downarrow}(\tau) = \frac{I_0[1 + (1-g)(\tau^* - \tau)]}{1 + (1-g)\tau^*} \,.} \tag{13.62}$$

From these, we can immediately find the cloud-top albedo

$$\boxed{r = \frac{I^{\uparrow}(0)}{I^{\downarrow}(0)} = \frac{(1-g)\tau^*}{1 + (1-g)\tau^*}, \qquad \tilde{\omega} = 1 \,,} \tag{13.63}$$

and the transmittance

$$\boxed{t = \frac{I^{\downarrow}(\tau^*)}{I^{\downarrow}(0)} = \frac{1}{1 + (1-g)\tau^*}, \qquad \tilde{\omega} = 1 \,.} \tag{13.64}$$

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The above expressions for r and t sum to one, as they must, since there is no absorption.

Not surprisingly, in the limit $\tau^* \rightarrow \infty$, the cloud-top albedo $r \rightarrow 1$, which also implies $t \rightarrow 0$. A bit more surprising, perhaps, is how large τ^* can become while still permitting significant transmission of radiation through a nonabsorbing cloud. For example, with $\tau^* = 100$, the transmittance t is still about 6%. All of this transmittance is associated with photons that have been scattered many hundreds of times on their journey through the cloud layer. It follows that if $\tilde{\omega}$ were even slightly less than one, the fraction of incident photons that would survive this journey would be substantially reduced.

Problem 13.4: A typical heavy stratocumulus cloud layer has an optical thickness $\tau^* = 50$, $\tilde{\omega} = 1$, and $g = 0.85$ in the visible band.

(a) Compute its albedo and total transmittance.

(b) If the cloud were perfectly absorbing rather than perfectly scattering, what optical thickness would yield the same transmittance as in (a), assuming $\overline{\mu} = 0.5$?

Problem 13.5: Repeat problem 7.11 but this time, for each case, compute the albedo. What is the difference in albedo for the two cases, and what does this difference suggest about the potential role of aerosol pollution in the global energy budget?

13.5 General Case

We previously considered the limiting cases of (i) a semi-infinite cloud ($\tau^* = \infty$) with arbitrary $\tilde{\omega}$, and (ii) a nonabsorbing cloud ($\tilde{\omega} = 1$) with arbitrary optical thickness τ^*. Last but not least, we may look at the more general case of (iii) arbitrary τ^* in combination with arbitrary $\tilde{\omega} < 1$.

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