(c) Examine Fig. 12.10 and determine the approximate range of wavelength over which the cloud *cannot* be considered 'semi-infinite.' Identify the associated spectral band(s).

13.6 Similarity Transformations†

In the expressions (13.65) and (13.66) that we derived for albedo and transmittance in the general case, there is an implicit dependence on *three* radiative quantities: the optical depth τ, the single scatter albedo $\tilde{\omega}$, and the scattering asymmetry parameter g. Yet all of this dependence is embodied in only *two* free variables: r_∞ and the product of Γ with τ^*. It follows that any two cloud layers having the same values of both r_∞ and $\Gamma\tau^*$ are *radiatively equivalent* to at least the accuracy of the two-stream approximation.

Because r_∞ itself is a function of both $\tilde{\omega}$ and g, *there is an infinity of combinations of these parameters that map to the same value of* r_∞. So if you were to measure the albedo at the top of a semi-infinite cloud and find, for example, that $r_\infty = 0.80$, you'd have no way of knowing whether you were dealing with $g = 0$ and $\tilde{\omega} = 0.988$, or with $g = 0.8$ and $\tilde{\omega} = 0.998$, or with any other combination that produces the same albedo. What you *can* uniquely determine, however, is the *similarity-transformed (or adjusted) single scatter albedo*, which is defined as

$$\boxed{\tilde{\omega}' \equiv \frac{1-g}{1-g\tilde{\omega}}\tilde{\omega}\ .} \tag{13.70}$$

It tells you what single scatter albedo *in combination with isotropic scattering* ($g = 0$) would give you the same r_∞ as your actual g and $\tilde{\omega}$. Note the effect of $g > 0$ (the usual case) is to make $\tilde{\omega}' < \tilde{\omega}$, except of course when $\tilde{\omega} = \tilde{\omega}' = 1$.

Similarly, the *similarity-transformed (or adjusted) optical depth* is defined as

$$\boxed{\tau' \equiv (1 - g\tilde{\omega})\tau\ .} \tag{13.71}$$

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It tells you what optical depth *in combination with isotropic scattering* ($g = 0$) *and adjusted single scatter albedo* $\tilde{\omega}'$ will give you the same value of $\Gamma\tau$ as would the "real" τ in combination with your cloud's actual $\tilde{\omega}$ and g. For $g > 0$ and $\tilde{\omega} > 0$, we find that $\tau' < \tau$.

> **Problem 13.8:** Verify the above interpretations of (13.70) and (13.71) by showing that $r_\infty(\tilde{\omega}', 0) = r_\infty(\tilde{\omega}, g)$ and that $\Gamma(\tilde{\omega}', 0)\tau' = \Gamma(\tilde{\omega}, g)\tau$.

The physical interpretation of (13.71) is straightforward. If g is close to unity, then scattered radiation will tend to continue more or less in the original direction of travel, almost as if it hadn't been scattered at all. Therefore, radiation will be able to traverse a greater optical depth without being absorbed than would be the case if g were smaller. The definition of τ' gives an "effective" optical depth that takes into account this phenomenon.

The interpretation of (13.70) is only slightly more subtle. The idea here is that if $g > 0$, then photons incident at cloud top will have a harder time "turning around" from their original downward path than would be the case for $g = 0$. On average, a greater number of scattering events will have to occur in order for a photon to have a good chance of re-emerging from the cloud top and contributing to the albedo. Of course, the greater the number of scatterings, the greater the fraction of photons that will be absorbed first, if $\tilde{\omega} < 1$. The definition of $\tilde{\omega}'$ takes into account the role of g in the overall absorptive properties of the cloud.

> **Problem 13.9:** Given $\tilde{\omega} = 0.99$ and $g = 0.85$, compute $\tilde{\omega}'$ and r_∞.

13.7 Clouds Over Non-Black Surfaces

In order to obtain our solutions (13.39) and (13.40) to the two-stream equations, we had to supply two boundary conditions. We chose to take $I^{\downarrow}(0) = I_0$ and $I^{\uparrow}(\tau^*) = 0$. The latter condition states that the lower boundary is perfectly black; i.e., there is no radiation incident on the cloud base from below.

If the lower boundary were in fact non-black, then radiation transmitted by the cloud would reach the surface, partially reflect upward and impinge on the cloud base from below. Some of that radiation would be transmitted back through the cloud. Some of the remainder would be reflect back downward, increasing the illumination of the surface, and so on, *ad infinitum*. The net result would be (a) an increase in the total downward flux incident on the surface and (b) an increase in the albedo at cloud top.

In principle, we could re-solve the two stream equations with a new lower boundary condition to account for a non-black surface. The new boundary condition would be $I^{\uparrow}(\tau^*) = r_{\text{sfc}} I^{\downarrow}(\tau^*)$, where r_{sfc} is albedo of the surface. But there is a simpler way, at least if we are content with finding the modified fluxes at the upper and lower boundaries.

Imagine that we have already used (13.65) and (13.66) to find the reflectivity r and total transmittance t of a cloud layer with specified τ^*, $\tilde{\omega}$, and g. The reflectivity and transmittance are intrinsic to the cloud itself, because there is no contribution from the lower boundary. The upward flux of radiation from cloud top is

$$F^{\uparrow}(0) = F_0 r \, , \tag{13.72}$$

where F_0 is the incident solar flux. The downward flux below cloud base is

$$F^{\downarrow}(\tau^*) = F_0 t \, . \tag{13.73}$$

Now let's place the same cloud layer over a surface with albedo r_{sfc}. Of the downward flux given by (13.73), a fraction r_{sfc} is reflected back toward the cloud. A fraction t of *that* is transmitted *through* the cloud, adding to the original $F^{\uparrow}(0)$ given by (13.72). An additional fraction r is reflected back downward toward the surface. A fraction r_{sfc} of *that* is reflected back upward, where it again contributes to an increase in $F^{\uparrow}(0)$ as well to the flux reflected back downward from cloud base.

Figure 13.10a depicts the first few terms in the infinite series of reflections between a cloud layer and the surface. The total upward flux at cloud top is now the sum of the cloud-reflected component

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