a black lower boundary. The second thing we notice is that if both t and r_{sfc} are *not* zero, then the second term on the right is greater than zero, implying an enhancement of the albedo relative to the original r. The third thing we notice is that if $t = 1$, then r must be zero (because $r + t + a = 1$); therefore $\tilde{r} = r_{\text{sfc}}$. The last case is of course equivalent to having no cloud at all. All of these inferrences make physical sense.

Recall that $\tilde{t}$ is the ratio of the flux incident on the surface below the cloud to the original flux F_0 incident at cloud top. If $t = 0$, then $\tilde{t}$ is also zero. If either r_{sfc} or r is zero, then $\tilde{t} = t$. Again, these results make sense.

But now consider the case that $r_{\text{sfc}} > 0$ and the cloud absorptance $a = 0$ so that $r = 1 - t$. We then get

$$\tilde{t} = \frac{1 - r}{1 - r_{\text{sfc}} r} \quad > \quad t = 1 - r \,. \tag{13.80}$$

Multiple reflections between the ground and the cloud thus enhance the downward flux at ground level, relative to what the flux would have been with a black surface. In other words, making the ground more reflective below you makes the sky brighter above you! If you live in a part of the country that gets snowfall, you will undoubtedly have noticed that an overcast sky is substantially brightened by the presence of snow on the ground.

More surprising, perhaps, is that if $r_{\text{sfc}} = 1$ and $a = 0$, then $\tilde{t} = 1$, implying that *the downward flux below cloud base is then exactly as large as it is above cloud top*, even when a large fraction of the incident radiation is reflected back to space before even passing through the cloud! In other words, in the absence of absorption, the presence of the cloud makes no difference whatsoever to the downward flux measured by an observer at the surface!

Although the above conclusion might seem counterintuitive, there is a simple physical explanation based on energy conservation: if there is no loss of radiative energy due to absorption in or below the cloud layer, then the upward flux of radiation below cloud base must increase through multiple reflection until just as much energy is lost by transmission upward through the cloud as is gained by downward transmission of the incident flux above the cloud. Since the transmittance of the cloud is the same in both directions, steady state is achieved when the two fluxes are equal. And since the upward flux below cloud equals the downward flux when the surface is perfectly reflective, the downward flux below cloud equals the downward flux above cloud.

Problem 13.11: The above argument can be equally well applied to the downward and upward flux (and thus intensity in the two-stream approximation) of radiation at an arbitrary level τ within a semi-infinite, nonabsorbing cloud. Specifically, we expect $I^{\downarrow}(\tau) = I^{\uparrow}(\tau) = I_0$.

(a) Outline the physical argument, drawing an analogy to the case of a finite cloud layer overlying a perfectly reflecting surface.

(b) Demonstrate the stated relationship, using the equations given earlier for the intensity in a semi-infinite layer.

(c) Demonstrate the stated relationship, using the equations given for the intensity in a nonabsorbing layer.

Problem 13.12: Assume that the incident flux of visible radiation on the top of a stratiform cloud layer is $F_0 = 400$ W m^{-2}. The cloud itself has a transmittance $t = 0.2$ and does not absorb. The surface albedo is initially $r_{\mathrm{sfc}} = 0.05$, but the cloud produces snowfall which blankets the surface, eventually raising r_{sfc} to 0.95.

(a) Create a table with two rows and six columns. The rows correspond to "before snowfall" and "after snowfall". The first three columns will contain, respectively, the downward flux, upward flux, and net flux at the surface. The last three columns will contain the same quantities, but at cloud top.

(b) By what percentage did the *downward* flux at ground level increase after the snowfall?

(c) For either case, compare the net flux at ground level with that at cloud top. Is one greater than the other? Why or why not?

13.8 Multiple Cloud Layers

We previously considered what happens when a cloud layer that partially reflects and partially transmits radiation is combined with

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a non-black (i.e., partially or totally reflective) surface. We can undertake a similar analysis to find the combined radiative properties of two cloud layers, the first with reflectivity r_1 and transmittance t_1; the second with r_2 and t_2. Figure 13.10b depicts the first few terms in the infinite series of reflections between the cloud layers. It can be shown that the total reflectance of the two-layer combination is given by

$$\boxed{\tilde{r} = r_1 + \frac{t_1^2 r_2}{1 - r_1 r_2}\,,} \qquad (13.81)$$

and the total transmittance is

$$\boxed{\tilde{t} = \frac{t_1 t_2}{1 - r_1 r_2}\,.} \qquad (13.82)$$

Problem 13.13: Write out the derivation for the above two equations.

Equations (13.81) and (13.82) apply to a combination of just two layers, but they can be used to compute the combined reflectance and transmittance of any number of layers. First, find the reflectance and transmittance of one adjacent pair of layers. Then treat this pair as a single layer to be combined with a third layer, and so on, *ad infinitum*.

Problem 13.14: Three nonabsorbing layers have transmittances $t_1 = 0.2$, $t_2 = 0.3$, and $t_3 = 0.4$.

(a) Compute their combined reflectivity and transmittance.

(b) Compare the computed transmittance with that predicted by Beer's Law for combinations of nonscattering layers, and explain the difference.

(c) Under what condition(s) is (13.82) consistent with Beer's Law, and why?

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